Smart Materials and Structures

PAPER

Acoustic-elastic metamaterials and phononic crystals for energy harvesting: a review

To cite this article: Guobiao Hu *et al* 2021 *Smart Mater. Struct.* **30** 085025

IOP Publishing

Smart Materials and Structures

Smart Mater. Struct. **30** (2021) 085025 (35pp)

https://doi.org/10.1088/1361-665X/ac0cbc

Acoustic-elastic metamaterials and phononic crystals for energy harvesting: a review

Guobiao Hu[1,2], Lihua Tang[2,*], Junrui Liang[1,*], Chunbo Lan[3] and Raj Das[4]

[1] School of Information Science and Technology, ShanghaiTech University, 393 Middle Huaxia Road, Shanghai 201210, People's Republic of China
[2] Department of Mechanical Engineering, University of Auckland, 20 Symonds Street, Auckland 1010, New Zealand
[3] State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing, People's Republic of China
[4] School of Engineering, RMIT University, GPO Box 2476, Melbourne, VIC 3001, Australia

E-mail: l.tang@auckland.ac.nz and liangjr@shanghaitech.edu.cn

Received 3 November 2020, revised 15 January 2021
Accepted for publication 18 June 2021
Published 6 July 2021

Abstract

Metamaterials and phononic crystals (PCs) with artificially designed periodic microstructures have attracted increasing research interests due to their unique properties that cannot be easily realized in natural materials. In recent years, the applications of metamaterials and PCs have been extended into the field of energy harvesting. A direct integration design strategy can yield a multifunctional system to suppress undesired vibrations/noise and convert them into electrical energy for providing power supply to widely distributed micro-electromechanical systems. Moreover, the defect state mode of metamaterials/PCs can localize the energy with an amplification effect, which has a great potential for improving energy harvesting efficiency. In addition, through tailoring the refractive index profile of metamaterials/PCs, the wave focusing phenomenon can be realized to boost the energy harvesting efficiency over a wide frequency range. This paper presents a comprehensive overview of the state-of-the-art advances in this direction over the past decade. According to the design strategies and working mechanisms, the existing studies in the literature on this topic are outlined and classified into three different categories. The advantages and disadvantages of various configurations are compared. The potential solutions to the existing drawbacks are discussed. An outlook on future prospects in this area is provided.

Keywords: metamaterials, phononic crystals, energy harvesting

(Some figures may appear in colour only in the online journal)

1. Introduction

Energy harvesting is the technology to extract electrical energy from ambient sources such as solar, thermal, wind and vibration etc. This field has been extensively investigated in the past few decades [1–5]. Energy harvesting is regarded as a promising solution for low power electronics to get rid of chemical batteries that need to be replaced or recharged constantly. In some cases where the replacement of batteries is costly and even not feasible (e.g. pacemaker, remote location sensors), energy harvesting technology provides a viable solution. As vibration-based energy harvesters take use of the most ubiquitous and accessible energy source, the research in this field has attracted immense interest in the past two decades. Conversion efficiency [6] and operating

[*] Authors to whom any correspondence should be addressed.

bandwidth [7] are two main concerns in the development of an energy harvester. Most of the previous research efforts have been devoted to improving the performance of energy harvesters in terms of these two aspects [8–11]. Other researchers have focused on achieving low resonant frequencies [12–16] and miniaturizing energy harvesters for compatibility with micro-electromechanical system (MEMS) manufacturing [12, 14, 15]. However, the performance of conventional harvester designs has been limited by the properties of natural materials and conventional structures.

In recent years, in the wake of the wide application of metamaterials and phononic crystals (PCs) in various disciplines, interests and attempts have also been dedicated to employing them for energy harvesting. PCs, which are conceptually similar to and inspired by photonic crystals [17, 18], are made of a periodic arrangement of scatterers embedded in matrices [19]. A remarkable trait of PCs is the band gap generated by the mechanism of Bragg scattering due to the periodic impedance mismatch [20, 21]. Metamaterials are artificial materials with intentionally designed internal micro-structures to achieve unique properties, such as negative refraction [22, 23], negative modulus [24, 25], negative effective density [26, 27], which are not commonly observed in natural materials. In the field of vibration energy harvesting, exploiting metamaterials and PCs as conversion medium for energy harvesting have been increasingly reported by researchers [28–30]. In particular, three review papers can be found on this topic [31–33]. However, the review article [31] was accomplished at the early stage of the development in this direction. Thus it does not cover recent new developments of metamaterial-based energy harvesting techniques. Moreover, the development in this direction grows much faster in recent years than at its early stage. As metamaterial-based energy harvesting has been increasingly attracted more attention in recent years. The review article [32] by Ahmed *et al* classified it as a relatively novel energy harvesting approach. However, the overview of metamaterial-based energy harvesting constituted only a subsection of that review article [32], which aimed to provide a comprehensive review of various energy harvesting approaches. On the other hand, though the review article [33] is a more recent informative review article which covers the themes of energy harvesting, metamaterials and Internet of Things, it lacks sufficient pertinence to the specific topic we are discussing. Therefore, there is a demand for a review paper that gives a comprehensive collection and summary of the literature on energy harvesting techniques based on metamaterials and PCs over the past decade.

Actually, converting vibrations into electrical energy is a bidirectional process. By introducing energy harvesters into metamaterials, mechanical energy is converted into electricity, and then dissipated or stored in a circuit. This kind of dissipation in the form of electrical energy is similar to the mechanical damping effect, through which the mechanical energy is directly transformed into heat [34, 35]. For example, shunted piezoelectric systems are often termed as piezoelectric shunt damping systems [36, 37] and they are widely used for applications in the field of noise/vibration control. In fact, through shunt circuits, not only the effective damping can be increased, but also, the elastic modulus of the piezoelectric element can be tuned [38]. With the implementation of the piezoelectric elements in the local resonators of metamaterials, the natural frequency of the local resonator, thus the band gaps can be controlled by tuning the shunt circuits [39–41]. Based on this mechanism, active tunable metamaterials can be realized. A recent review article provided a comprehensive overview of the existing techniques on the specific topic of active tunable metamaterials [42]. The present review article aims to summarize the existing research on the opposite side: how metamaterials and PCs serve for energy harvesting purposes.

The rest of this review is organized as follows. Section 2 introduces the basic concepts and fundamentals of energy harvesting, PCs, and metamaterials. Subsequently, based on the system features and the underlying design mechanisms, we classify all the existing configurations into three categories: directly integrated multifunctional configuration, defect state based configuration, and wave focusing mechanism enhanced configuration. Sections 3–5 present the overview of recent advancements in each direction, respectively. In section 6, we summarize and compare all the design approaches in the literature. We retrospect the advantages and disadvantages of each type of configuration to provide some guidelines for readers interested in this direction. Potential solutions to the drawbacks of the existing configurations are briefly discussed. Future prospects that deserve more research attention are proposed.

2. Basic concepts and fundamentals

2.1. Vibration energy harvesting

Vibration energy harvesting has attracted immense research interests in the last decade [43–46], as this technology has a promising potential for powering widely utilized low-power electronics. Since vibration-based energy harvesters take use of the most ubiquitous and accessible energy source and have wide potential applications, the research in this field has attracted tremendous interests from both academia and industry.

2.1.1. Energy transduction mechanisms. Three main energy conversion mechanisms used for vibration energy harvesting are electromagnetic [47], electrostatic [48], and piezoelectric [49, 50] transductions. The electromagnetic transduction mechanism refers to Faraday's law of induction, which states that a current will be induced in a coil due to any change of the magnetic environment in which the coil is placed [51]. An electromagnetic vibration energy harvester mainly consists of three components: a coil, a magnet, and an oscillator [52]. Either the coil or the magnet is attached to the oscillator. When the external excitation forces the oscillator to vibrate, a relative displacement is generated between the coil and the magnet. A current is induced in the coil due to the change of the magnetic flux through the coil. The electrostatic transduction mechanism employs a parallel-plate variable capacitor with a pre-charged voltage or constrained charge on the electrodes. According to the formula of an ideal parallel-plate capacitor

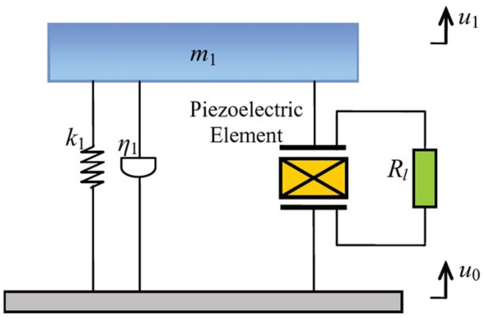

Figure 1. SDOF piezoelectric energy harvester [56].

[43], when the external vibration is exerted on one plate of the capacitor and forces the gap of the parallel plates to vary, the effective capacitance of the capacitor varies [53]. For the pre-charged voltage mode, charges flow in and out of the capacitor through a circuit to stabilize the pre-charged voltage and deliver electrical energy into the load in the circuit.

Piezoelectric transduction is the most commonly used energy conversion mechanism due to the high power density and high energy conversion efficiency [54]. Most of the existing metamaterial-based energy harvesters are realized using piezoelectrics. Therefore, this subsection provides a brief retrospect of the fundamentals of piezoelectric energy harvesting. Piezoelectric materials can generate an electric voltage in response to an applied external force and vice versa. The relationship between a mechanical stress and an electrical voltage in piezoelectric materials is termed the piezoelectric effect which can be basically described by the constitutive equations as follows [49, 55]:

$$\begin{aligned} S &= T/Y + dE \\ D &= dT + \varepsilon E \end{aligned} \tag{1}$$

where T is the mechanical stress, S is the strain, Y is the modulus of elasticity, ε is the dielectric constant, d is the piezoelectric constant, E is the electric field, and D is the electric displacement.

The simplest way to describe a piezoelectric energy harvester is by using the lumped parameter model. The schematic of a single-degree-of-freedom (SDOF) piezoelectric energy harvester subjected to the base excitation of u_0 is shown in figure 1 [56]. The governing equations of this system can be written as:

$$\begin{cases} m_1\ddot{u}_1 + c_1(\dot{u}_1 - \dot{u}_0) + k_1(u_1 - u_0) + \theta V = 0 \\ \frac{V}{R} + C_p\dot{V} - \theta(\dot{u}_1 - \dot{u}_0) = 0 \end{cases} \tag{2}$$

where m_1, k_1, and c_1 are the mass, stiffness, and mechanical damping of the system, respectively; $C_p = \varepsilon_{33}^S A/t$ is the clamped capacitance of the piezoelectric element. When the piezoelectric transducer is used in the 3–3 mode, the electromechanical coupling coefficient is defined as [57–61]: $\theta = e_{33}A/t$. A and t are the surface area normal to the three-direction and the thickness in the three-direction of the piezoelectric transducer, respectively. e_{33} and ε_{33}^S are the piezoelectric constant and permittivity, respectively. R is the electrical resistance connected to the piezoelectric transducer.

2.1.2. Energy harvesting enhancement from structural design perspective. Since most ambient vibration energy spreads over a broad spectrum, operating bandwidth is one of the main concerns in designing vibration-based energy harvesters. In order to achieve broadband energy harvesting, various techniques have been proposed for broadening the bandwidth. Generally speaking, most broadband energy harvesting techniques can be realized through the optimization of mechanical structures. Some common methods from the existing literature are summarized and normally classified as follows.

2.1.2.1. Multi-mode technique. Traditional linear energy harvesters are designed to work in resonance. The energy harvesting performance will degrade significantly if the external excitation frequency shifts away from the resonant frequencies of energy harvesters. Though a traditional resonant-type vibration energy harvester has multiple resonant frequencies, the voltage output at higher-order resonances is much smaller than that at the fundamental resonance [62]. Therefore, only the first mode is useful for energy harvesting, and the operating bandwidths of traditional energy harvesters are very narrow. To overcome this limitation, multiple DOF energy harvesters with useful vibration modes close to each other and comparable outputs have been proposed by researchers to cover a broader range of frequency for efficient energy harvesting [63, 64].

2.1.2.2. Nonlinear technique. Multi-modal energy harvesters can produce wider operating bandwidths, which is usually achieved with the increase of the volume and the weight of the devices; however, the energy density is sacrificed. In the past decade, nonlinear oscillators have been explored for broadband energy harvesting [65, 66]. The bending phenomenon of the amplitude-frequency response curve around resonance due to the nonlinearity results in a wider range of frequency for energy harvesting [67, 68]. With properly designed system parameters, the internal resonance of the nonlinear system could be realized to give further extended bandwidth with more bent resonant peaks than those in the traditional nonlinear design [68, 69].

2.1.2.3. Resonant frequency tuning technique. Resonant frequency tuning enabled energy harvesters were designed to detect and track the frequency variation in external excitation. They passively or actively tune the resonant frequency to match the excitation frequency for maximizing power output in a wide frequency range [70, 71]. The passive techniques do not need any additional energy to implement the tuning [72]. The active tuning requires extra power input to achieve frequency detection and tuning, which might significantly sacrifice the harvested power; therefore, it is only suitable for the situation where the duty cycle for frequency tuning is really low.

2.1.3. Energy harvesting enhancement from circuit design perspective. In real applications, the alternating current that directly flows out of the energy harvester usually needs to be rectified and conditioned before being delivered to power devices. Therefore, the design and analysis of a practical energy harvesting system should take into account both mechanical and electrical aspects. On the other hand, the conversion efficiency is another important figure of merit to assess of the performance of an energy harvester. Electrical parts (i.e. interface circuits) play an important role in realizing efficient energy harvesting. Using advanced interface circuits, such as synchronized charge extraction (SCE) [73, 74], synchronized switch harvesting on inductor (SSHI) [75–78], can boost the power outputs of the energy harvesters. A study showed that a highly efficient P-SSHI interface circuit could increase the power output for about 5.8 times [79]. However, it is worth noting that using advanced interface circuits can significantly boost the conversion efficiency only under the weak coupling condition [60, 61]. Research has proved that the power limit of a piezoelectric energy harvester, is determined once the mechanical structure is manufactured [80]. To be more specific, the power limit of an energy harvester with a given mechanical structure is dependent on the mechanical damping of the system. Using electro-mechanical analogies, the electrical equivalent of the damping coefficient is the resistance. By considering the equivalent resistance of the damping coefficient as the internal resistance, the power limit can be obtained only when the external resistance satisfies the impedance matching condition [81, 82]. Interested readers in energy harvesting interface circuits could refer to the review paper [83] that has collected and summarized the relevant literature on this topic.

2.2. Phononic crystals and metamaterials

PCs and metamaterials are both periodic structures but based on different operation principles, that is, Bragg scattering effect and local resonance effect, respectively. The study of PCs emerges from the end of the last century as a branch of the research of periodic materials and structures [84]. The realization of PCs can be achieved in different forms, e.g. beams, rods, plates. The applications of PCs based on the band gap behaviour include vibration suppression [85], noise reduction [86, 87], and wave collimation [88] etc. The research of metamaterials is relatively a bit later and starts after a seminal paper published in 2000 by Liu *et al* [89] that introduced a first conceptual realization. In fact, metamaterials can be classified into two categories: electromagnetic metamaterials and acoustic-elastic metamaterials, according to the types of waves on which metamaterials are engineered to manipulate. Acoustic-elastic metamaterial is the mechanical analogy of and actually originated from the electromagnetic metamaterial. The unique properties derived from the micro-structures rather than the properties of the base materials give metamaterials a promising future for applications in many fields such as noise/vibration control [90, 91], cloaking [92], superlens [93] etc. Due to the different working mechanism, metamaterials can break through the size limitation of PCs and generate sub-wavelength band gaps [21]. Therefore, metamaterials have attracted comparatively more research interests in recent years.

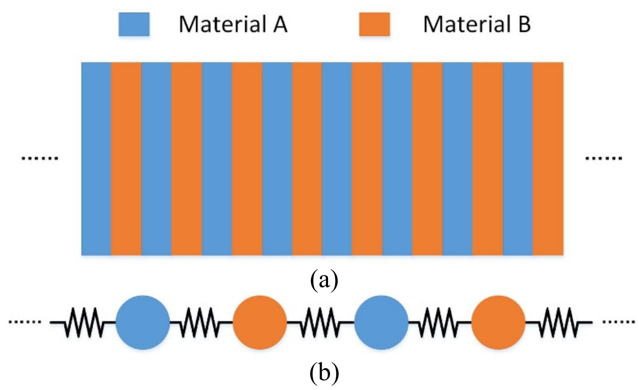

Figure 2. One-dimensional PC (a) physical model; (b) mass-spring model.

Though numerous studies in the field of PCs and metamaterials have been reported in recent years, there still lacks a clear and universally accepted definition of metamaterial [21]. The term acoustic-elastic metamaterial is used by some researchers to in particular refer to the concept introduced by Liu *et al* [89], while some others think that PC which has a history tracked back to early 1990s should also be included within the concept of metamaterial [21]. In contrast, in some other literature [19, 94], the acoustic-elastic metamaterial is broadly considered as an extension/evolution of PC, due to it exhibiting a similar band gap phenomenon for wave transmission. To differentiate them, Bragg scattering PC and locally resonant PC are also often used by researchers [21].

To unify the terminologies throughout this paper, hereinafter, 'phononic crystal' refers to the conventional PC based on Bragg scattering effect (for short, Bragg scattering phononic crystal or PCs) and 'metamaterial' refers to the acoustic-elastic metamaterial based on local resonance effect (for short, locally resonant metamaterial or locally resonant PC). This section briefly reviews the band gap generation mechanisms of PCs and metamaterials. The advantages and disadvantages of PCs and metamaterials are compared and discussed.

2.2.1. Phononic crystals. Bragg scattering PCs, or for short, PCs [95] usually consist of two (or more) kinds of materials with different properties (such as mass density, bulk modulus etc.). The schematic of one-dimensional PC is shown in figure 2. The stiff/dense material forms the substrate.The compliant/light material is often used as inclusions. Due to the Bragg scattering effect caused by the mechanical impedance mismatch and the periodical distribution of inclusions, the band structure (i.e. dispersion relation) is changed. Therefore, the band gap phenomenon appears in the band structure of the PC. Acoustic/elastic waves of frequencies that fall into the band gap are insulated and attenuated. Due to this band gap phenomenon, PCs are very suitable to be used in applications for noise/vibration control. Obviously, the noise/vibration suppression ability of the PC strongly depends on the width of the band gap.

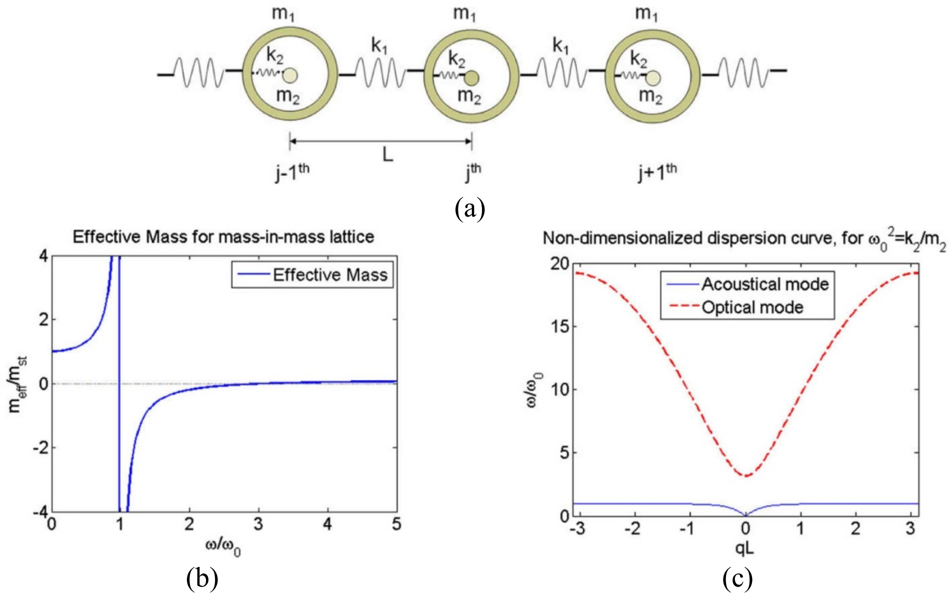

Figure 3. (a) Mass-in-mass model representation of metamaterial, (b) effective mass of a unit cell, (c) nondimensionalized dispersion curve for the mass-in-mass lattice model [27]. Reprinted from [27], Copyright (2009), with permission from Elsevier.

Investigations on the band gap of PCs have been widely reported, including band structure computing techniques and numerous research attempts to widen the band gap or make it tuneable. To calculate the band structure, the transfer matrix method (TMM), the plane-wave expansion methods [95–97], the finite difference time domain method, and the multiple scattering theory, and the lumped mass method have been developed. A detailed introduction of these methods can be found in [98]. The band structure can be tailored through tuning the filling ratio [95, 97, 99] and using new materials with different elastic properties [100]. Piezoelectric PC, which contains piezoelectric materials, can have a wider band gap due to the tunability through the piezoelectric effect [97, 101, 102].

2.2.2. Metamaterials. The locally resonant metamaterials, or for short, metamaterials, also have periodically distributed inclusions embedded inside of its substrate and they are formed by at least three kinds of materials. The third kind of material between the inclusions and substrate is relatively soft and used to enwrap the inclusion. In this manner, the inclusion and the outside coating layer can be regarded as a local spring-mass oscillator system. The local oscillator will exhibit out-of-phase motion during the vibration near its resonance frequency. These local oscillators can be essentially regarded as a series of periodic vibration absorbers tuned to the forcing frequency of the vibration source. Every local oscillator of the metamaterial gradually neutralizes the input excitation through the out-of-phase motion near its resonant frequency. After several unit cells, the vibration energy transferred from the vibration source can be significantly reduced. Thus, wave propagation in the substrate is banned, which is another band gap mechanism that can be used for vibration control and noise insulation. For PCs, band gap phenomenon occurs only when the wavelength and the distance between inclusions, namely, the lattice constant, satisfy a specific relationship for generating the Bragg scattering effect. While the band gap generation of metamaterials is independent of the lattice constant and it is only related to the natural frequency of the local oscillator. This makes it easier to customize the band gaps of metamaterials. Since metamaterials do not have any dimension limitations, they are more effective in attenuating and filtering low-frequency vibrations [103, 104].

One-dimensional metamaterials with frequency-dependent effective mass, which is defined as the ratio of momentum to velocity [105], can be mathematically modelled as a chain system of springs and masses. The spring-connected outer masses m_1 having an inner mass m_2 embedded inside suspended by a spring with constant k_2 [27, 106]. The schematic of the model is shown in figure 3(a). When the excitation is near the resonance frequency of the local resonance, the out-of-phase motion of the inner mass leads to the whole unit exhibiting a negative effective mass (figure 3(b)). A typical metamaterial can generate a band gap, as illustrated in figure 3(c). Profiting from this property, an important application of this kind of metamaterial is for vibration suppression. Some literature explained that the band gap originates from the negative effective mass effect and thus concluded that the band gap occurs in the range of negative effective mass. In most circumstances, the above statement is valid. However, strictly speaking, there is a subtle difference between the two concepts, namely, the negative effective mass range and the band gap range. A rigorous mathematical derivation shows that the band gap range is not completed confined within the negative effective mass range [107].

As the equivalent counterpart in the acoustic domain, an acoustic metamaterial shown in figure 4 can be obtained using an array of Helmholtz resonators which could be analogized to lumped mass-spring local resonators and further to inductor–capacitor resonant circuits [108]. Near the resonant frequency of the Helmholtz resonator, the acoustic pressure at the neck

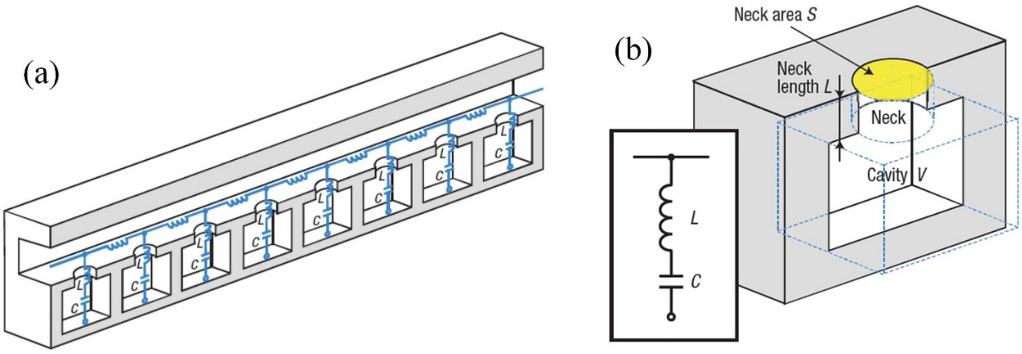

Figure 4. (a) Schematic of an acoustic metamaterial consists of an array of Helmholtz resonators, (b) illustration of the analogy between a Helmholtz resonator and an inductor–capacitor circuit [108]. Note that an inductor–capacitor circuit is the electrical analogy of a mechanical mass-spring oscillator. Reprinted by permission from Springer Nature Customer Service Centre GmbH: Springer Nature, natural materials [108], [COPYRIGHT] (2006).

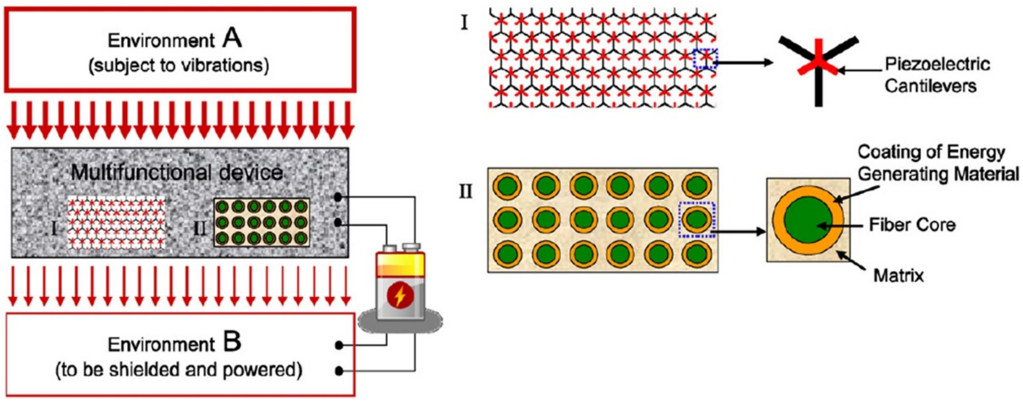

Figure 5. Multifunctional metamaterial design based on honeycomb structure embedded with piezoelectric cantilevers [109]. Reprinted from [109], Copyright (2009), with permission from Elsevier.

of the Helmholtz resonator is out of phase with the acoustic pressure in the main duct, resulting in the reciprocal cancellation and thus the reduction of sound transmission in the main duct.

3. Directly integrated multifunctional configurations

By integrating energy harvesters into metamaterial systems, the vibration energy existing in the systems can be harvested. Moreover, the energy harvesting process can effectively increase the system damping, which, in return, benefits vibration suppression. Therefore, metamaterial systems integrated with energy harvesters are expected to have dual-functionalities. In the following, we divide this section into two subsections. The first subsection summarizes the development of multi-functional elastic metamaterial energy harvesting systems. The second subsection overviews the existing multi-functional acoustic metamaterial energy harvesting systems in the literature.

3.1. Elastic wave energy harvesting

Gonella *et al* [109] first explored the idea of achieving a multifunctional system (as shown in figure 5) by combining the design of both metamaterials and energy harvesters. One application configuration of the proposed system is based on a hexagonal truss-core honeycomb. Because of the periodicity of the microstructure, the hexagonal truss-core honeycomb can produce Bragg-scattering band gaps. Moreover, the introduction of the piezoelectric cantilevers also brings the band gap due to local resonance in the honeycomb. On the other hand, the introduced piezoelectric cantilevers can convert mechanical stress into electrical voltage. They examined the power output at frequency points immediately before and after the local resonance induced band gap. This pioneering work devoted most effort to investigating the band gap behaviour of the proposed system while less effort in evaluating the energy harvesting performance. In addition to the multi-functionalities, another important advantage being remarked in [109] is that due to the metamaterial concept-based design, the embedded piezoelectric energy harvesters can be shielded from the damage caused by vibrations and shocks. However, this statement is only valid when the proposed system operates near the band gaps. From the energy harvesting perspective, to maximize the power output, the system is required to operate near the resonance in which the shielding capability of the proposed system will not work.

The mass-in-mass model has been widely employed for the investigation of metamaterials. The mathematical treatment

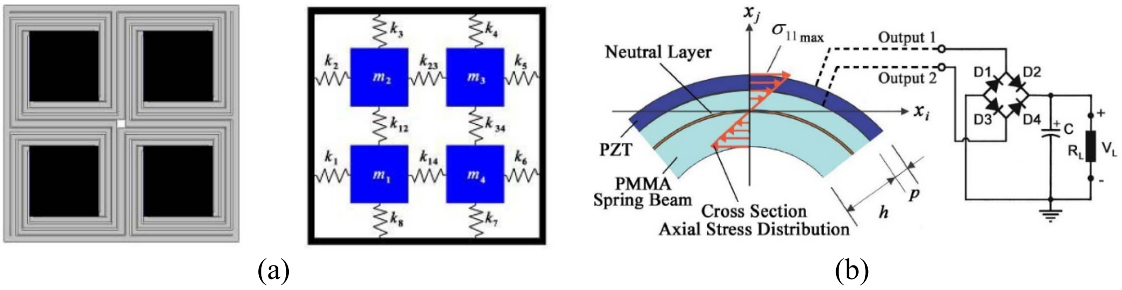

Figure 6. (a) A unit cell of the metamaterial in [111], and (b) its implementation of piezoelectric shunting circuit for energy harvesting. Reproduced with permission from [111].

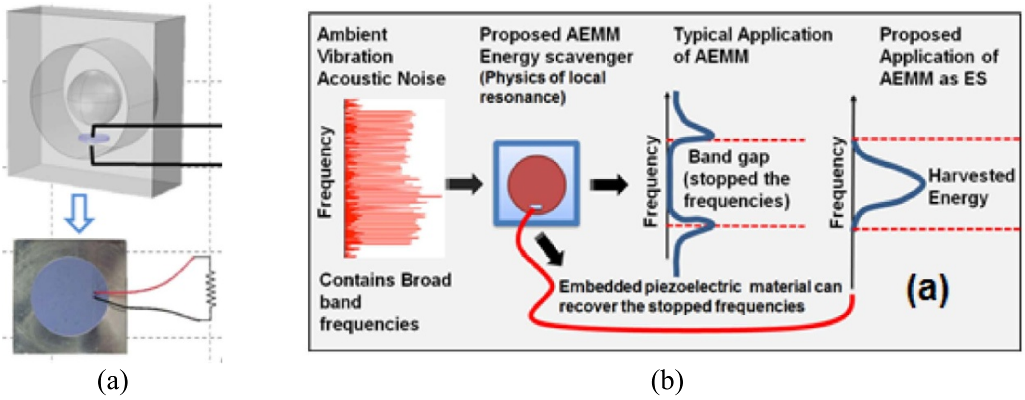

Figure 7. (a) Schematic of a unit cell of the AEMM based energy harvester, (b) the depicted working mechanism [112]. Reproduced from [112].

of the lumped parameter model reflects the intrinsic mechanism of metamaterials. Therefore, it is intuitive to design structures based on the mass-in-mass model. Mikoshiba *et al* [110] proposed an energy harvesting system with a periodic structure embedded with multiple local resonators made of spring-suspended magnets. The inner mass is a cylindrical neodymium magnet that can be used together with the wrapped coils to constitute a readily prepared electromagnetic generator. When the inner mass (i.e. the magnet) oscillates, the magnetic fields of the magnets induces the current in the coils. They derived the dispersion relation and predicted the band gap of the proposed metamaterial energy harvester. A simple experimental study validated the predicted band gap. The experimental results showed that the prototyped device could generate a peak voltage of 3.03 V with an output power of 36 mW across a 1 Ω load under the harmonic excitation, close to the resonant frequency of the local resonator.

Zhang and Wu [111] conceptually designed a metamaterial with local resonators realized using folded beams as shown in figure 6(a). The designed metamaterial can be equivalently explained using the mass-in-mass model. Each unit cell contains multiple local resonators and the theoretical calculation proved the existence of multiple band gaps in the low-frequency range from 40 to 250 Hz. The modal analysis revealed that the vibration energy was localized in the folded beam for some modes of metamaterial, which had the potential for energy harvesting. To realize the idea of energy harvesting, they attached piezoelectric transducers on the folded beams, as illustrated in figure 6(a). They performed a simulation to evaluate the stress variation in the folded beams and indirectly predicted the voltage and power output from the conceptually designed metamaterial energy harvester using a similar stress–voltage relationship as presented in [109].

Ahmed *et al* [112, 113] also designed an energy harvester based on a unit cell of metamaterial, as shown in figure 7(a), with the original intention for harnessing acoustic energy. Since they only tested the energy harvesting performance, in fact, under a base excitation, this study is classified into the elastic wave energy harvesting type research. They commented that due to the local resonance, the dynamic energy could be entrapped inside the local resonator, which can thus be recovered using the energy harvesting technique. In addition, they argued that the maximum power could be generated in the band gap when the acoustic energy is entrapped as depicted in figure 7(b). However, in the analytical calculation, the forced vibration of an SDOF oscillator is considered. Moreover, the prototype is excited by a shaker instead of acoustic excitation in their experiment. In addition, the matrix is made of a thick aluminum block and can be regarded as a rigid body in the low-frequency range of concern. These treatments indicate that their conclusions are all obtained based on the analysis of an SDOF resonator. The similar statement also appeared in [114]. However, the statement that the maximum power can be produced in the band gap is not always valid. As some early exploration work, they did not clearly explain the conditions under which the above statement is valid. A later research by

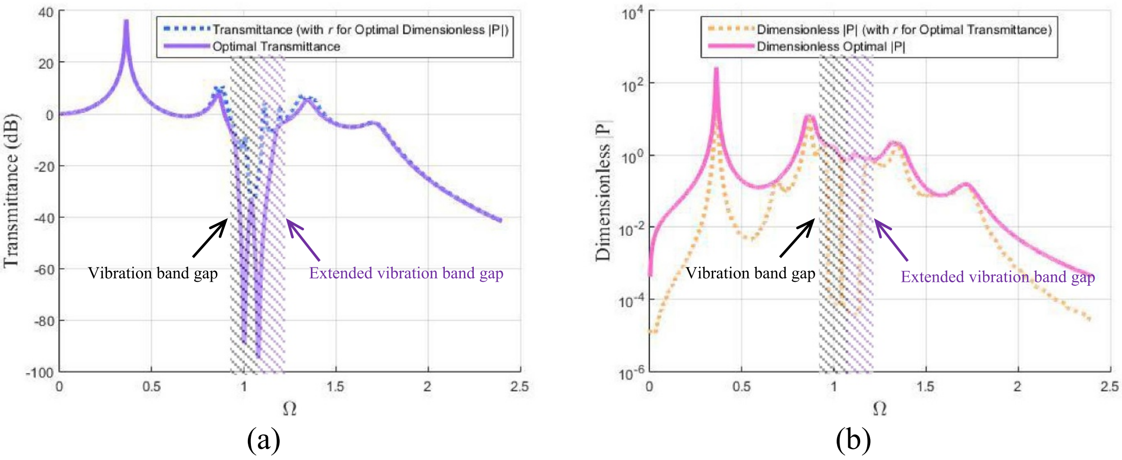

Figure 8. (a) Transmittance and (b) dimensionless power output of the metamaterial-based energy harvester presented in [115]. Reproduced with permission from [115].

Hu *et al* [115] pointed out that the harvested energy inside the band gap is very minor due to the very weak vibration intensity of the whole structure, as shown in figure 8. The maximum power could only be achieved near the resonance of the structure as a whole when the vibration intensity reaches the maximum. Based on the similar design presented in figure 7(a), Mir *et al* [116] and Saadatzi *et al* [117] explored the feasibility for simultaneous energy harvesting and noise insulation through finite element simulation. However, their finite element results could not strongly support their statements.

Beams and plates are widely used in engineering designs. Vibration suppression of these structures is critically important. On the other hand, numerous vibration energy harvesters are designed based on beam structures. For these reasons, designs of multi-functional metamaterials based on a beam or plate-like structures emerge [94, 118–122]. Chen *et al* [118] studied a piezoelectric phononic beam consisting of a cantilever beam periodically bonded with piezoelectric transducers and ballasted by a series of rigid masses. The phononic beam can only produce Bragg scattering band gaps. Both the analytical and finite element models are developed to predict the band gaps of the piezoelectric phononic beam. However, these Scattering band gaps were opened at high-frequency range. An experiment was then conducted to validate the band gaps and also evaluate the energy harvesting performance of the piezoelectric phononic beam under a broadband white noise signal excitation. Shen *et al* [94] designed a metamaterial beam with an array of spiral beams as springs for local resonators as well as energy transduction medium. It is worth mentioning that spiral beams have larger effective lengths and can achieve lower natural frequencies than conventional straight beams, thus beneficial for opening low-frequency band gaps and realizing low-frequency energy harvesting [123]. As shown in figure 9(a), the piezoelectric transducers are bonded at the root of the spiral beams. In the experimental study, the voltage outputs from the piezoelectric transducers were evaluated at the resonant frequencies of the metamaterial beam. It was reported that in the low-frequency range of 0–500 Hz, the designed metamaterial beam energy harvester could produce voltage outputs with amplitudes more than 1.2 V at a dozens of resonant frequencies. Using mass-loaded membrane type resonators, Chen *et al* [124] also designed a metamaterial beam for suppressing transverse vibration and harvesting energy. Because of the split ring design, the membrane exhibited two resonant modes with close eigen-frequencies. Thus, the metamaterial beam produced two band gaps close to each other for broadband vibration suppression. They claimed that the frequency where the maximum voltage output was generated coincided with the maximum vibration attenuation frequency. This is because they only measured the voltage output from a single PVDF patch attached to a single membrane. Since the membrane used for energy harvesting was near the clamped root of the host beam, the effective bending stiffness of the small segment of the host beam was very large. The membrane underwent almost an independent uncoupled motion. Hence, the maximum voltage could be generated near the resonant frequency of the membrane. However, if the PVDF patch was attached with the membranes near the tip of the host beam, the maximum voltage could never be generated near the resonant frequency of the membrane itself, since the membrane together with the host beam formed new resonant modes at different frequencies.

Hu *et al* [121] proposed a metamaterial beam with internally coupled local resonators, as shown in figure 9(b). The small cantilever beams serve as the local resonators. The piezoelectric transducers are attached to the local resonators. The neighbouring local resonators are alternately connected through springs. Both vibration suppression behaviour and energy harvesting performance of the proposed system were studied based on the derived analytical model. Thanks to the coupling springs, the proposed metamaterial beam produced two band gaps in the low-frequency range. As compared to the means by adding extra local resonators to open additional band gaps [126], the method proposed in [121] does not increase the weight of the entire structure. While, the drawback is that the internal coupling is not easy for physical implementation. On

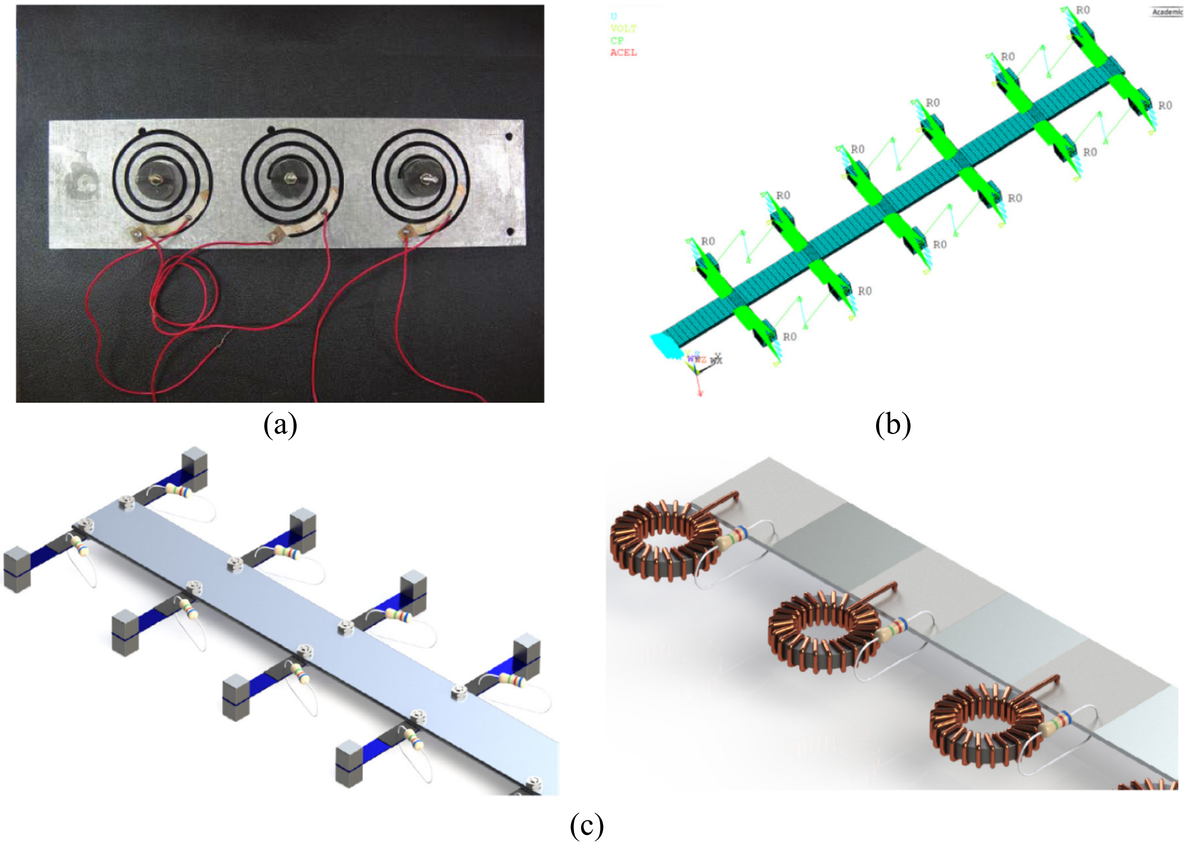

Figure 9. (a) A piezoelectric metamaterial with spiral beam structures [94], (b) a piezoelectric metamaterial beam with internal couplings through spring connections [121] (c) a conventional metamaterial beam embedded with piezoelectric transducers (left) and an electromechanical metamaterial with energy harvesting capability (right) [125]. Reproduced with permission from [94]. Reprinted from [121], with the permission of AIP Publishing.

the other hand, regarding the energy harvesting performance, it was found that the voltage output of the internally coupled piezoelectric metamaterial can be increased by four times as compared to the design based on a conventional metamaterial beam in [119]. In terms of the modelling of piezoelectric material covered metamaterial beams, Hu *et al* [122] then derived a one-dimensional piezoelectric composite finite element. Using the derived element, they presented several case studies of different piezoelectric metamaterial systems and analysed both the band gap behaviour and the energy harvesting performance.

Based on a conventional metamaterial beam similar to that presented in [119], Sugino and Erturk [125] and [120] modelled and analysed an energy harvesting meta-structure as shown in figure 9(c). Instead of using the conventional TMM method and Bloch's theorem, they developed a novel method for estimating band gaps of metamaterial beams based on modal analysis. In this method, with the assumption of an infinitely number of local resonators, a cumulative term in the transfer function can be simplified by the integral term of the orthogonality condition. They also analytically investigated the energy harvesting performance and concluded that the integrated energy harvesters could produce useful energy and did not influence the vibration suppression of the metamaterial. In addition, both piezoelectric and electromagnetic transductions were considered in the conceptual design of the local resonators for energy harvesting, as shown in figure 9(c). Studies on similar configurations or based on similar mechanisms are not detailed anymore but can be referred to [127–136].

The design of an energy harvester involves two aspects: mechanical structure and electrical interface circuit. In the existing literature on metamaterial-based energy harvesting, most work focused on the complicated mechanical structure and simplified the interface circuit as a purely resistive load. A recent work by Chen *et al* [137] reported a piezoelectric metamaterial plate shunted to a self-powered SCE circuit (SP-SCE) for energy harvesting. The mechanical structure and the electric interface circuit, shown in figures 10(a) and (b), are modelled based on the Kirchhoff plate theory and the equivalent impedance method [138], respectively. The results showed that shunting to the SP-SCE interface circuit has few effects on the band gap behaviour of the metamaterial plate, while shunting to a simple inductor can generate a new dispersion curve and broaden the first band gap for benefiting low-frequency vibration suppression. In terms of the voltage output, the SP-SCE did not boost the power output due to the low piezoelectric voltage. After inserting an inductor in parallel with

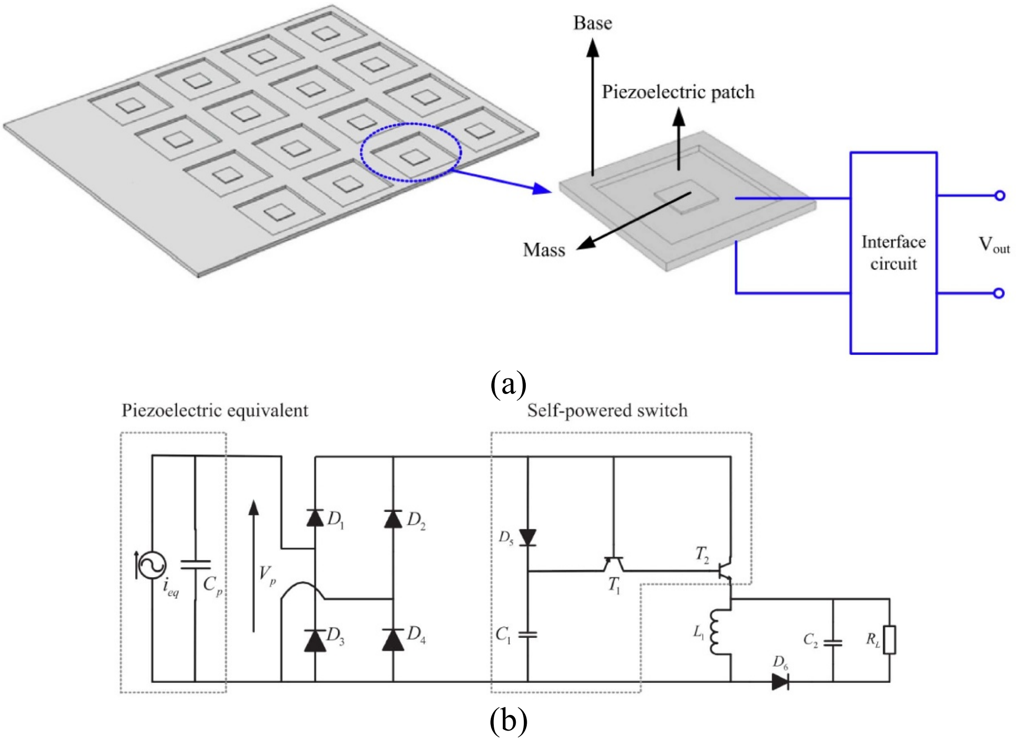

Figure 10. (a) Mechanical structure of a piezoelectric metamaterial plate shunted to (b) the SP-SCE circuit [137]. Reprinted from [137], Copyright (2020), with permission from Elsevier.

the piezoelectric capacitor, the output voltage amplitude of the piezoelectric transducer was increased for about 200%.

3.2. Acoustic wave energy harvesting

The studies reviewed in section 3.1 focused on harvesting energy from vibrations in elastic metamaterials. Li *et al* [139] designed an acoustic metamaterial device that was capable of both sound insulation and energy harvesting, as shown in figure 11. The pre-stressed thermoplastic polyurethane membrane together with the rigid mass constitute the local resonator. They experimentally evaluated both the noise insulation and the energy harvesting performance of the dual-functional acoustic metamaterial. A similar energy harvesting system based on the membrane-type metamaterial was reported in [140]. Related in-depth modelling of a membrane-type local resonator can found in [141, 142]. The acoustic local resonator can be simplified and becomes equivalent to a mass-spring oscillator [143].

Wang *et al* [144] designed a dual-membrane coupled acoustic metamaterial and inserted a piezoelectric transducer inside the cavity between the two membranes. Figure 12(a) shows the schematic of a unit cell of the proposed acoustic metamaterial. Each membrane is equivalent to an oscillator connected to the ground (i.e. the holder shown in figure 12(a)) and the air in the cavity between the two membranes plays the role as an air spring that couples the motions of the two membranes, i.e. two oscillators. Hence, the dual-membrane coupled system can be understood as a coupled 2-DOF system with two resonant modes which has been confirmed by their simulated results

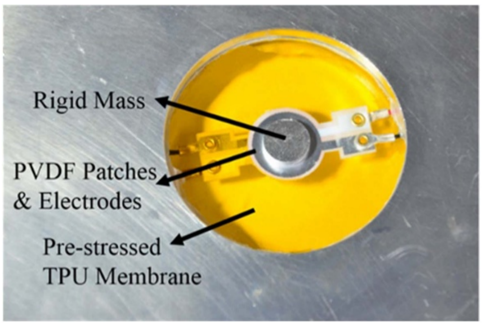

Figure 11. The prototyped acoustic metamaterial with both noise insulation and energy harvesting capabilities [139].

(figure 12(b)). When the resonance takes place, the sound pressure inside the cavity is expected to be amplified for improving the energy harvesting efficiency. An experimental study validated that an amplification of about 4.2 times could be achieved when the proposed dual-membrane coupled acoustic metamaterial was used for energy harvesting.

The Helmholtz resonator is a very classic acoustic system that can often be regarded as a single degree of freedom acoustic oscillator. Zhang *et al* [145] employed the Helmholtz resonator in the design of acoustic metamaterials. Actually, as shown in figure 13, the acoustic resonator is a combination of a Helmholtz resonator and a membrane. The rigorous mathematical modelling of such kind of system can be found in [143]. Due to the acoustic-structure coupling between the Helmholtz resonator and the membrane, the designed acoustic

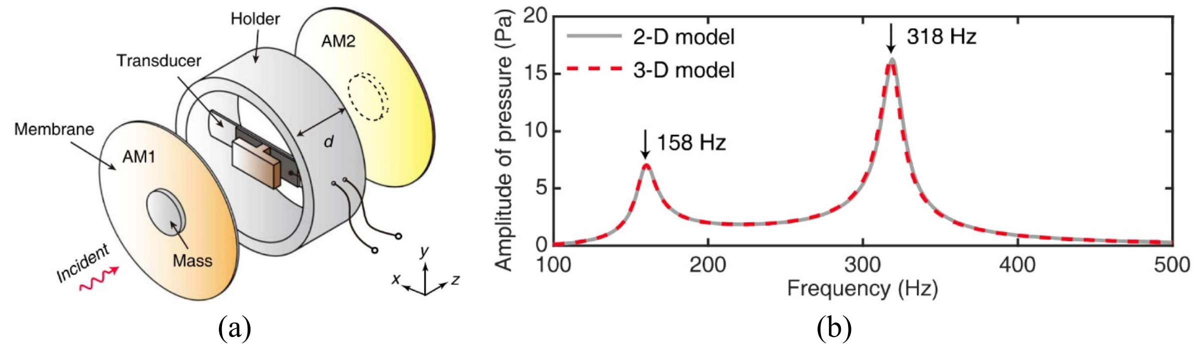

Figure 12. (a) Dual-membrane coupled acoustic metamaterial with piezoelectric transducer for energy harvesting [144], (b) simulated sound pressure response inside the cavity between the two membranes.

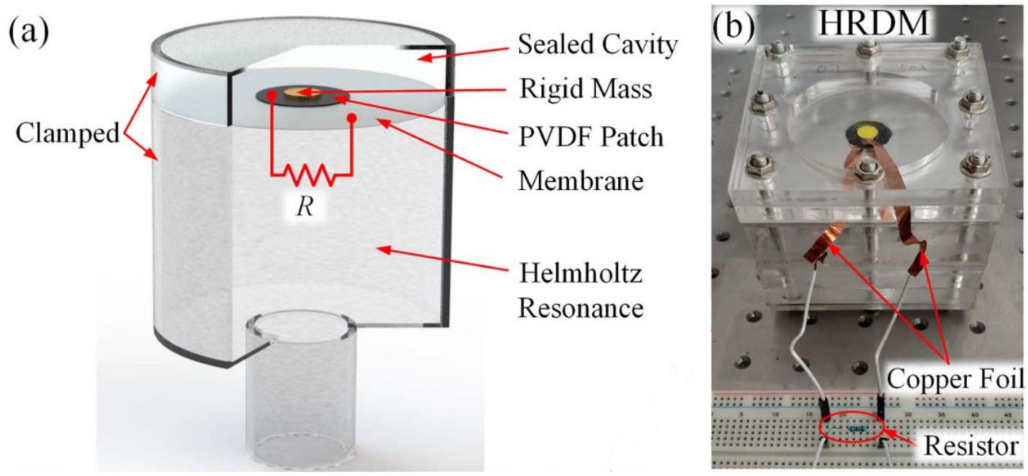

Figure 13. (a) The illustration of the structure of the acoustic metamaterial proposed in [145], (b) the fabricated physical prototype used in the experimental study.

resonator can be considered as a 2-DOF system in the low-frequency regime. Therefore, two sound insulation bands were obtained. Two resonant frequencies could be used for energy harvesting. It is worth mentioning that sound insulation peaks and power peaks are located at the same frequencies, which means that both noise reduction and energy harvesting can be truly realized simultaneously. This result seems to be contrary to the conclusion from elastic wave energy harvesting using metamaterial and PCs as summarized in the previous subsection.

In fact, both conclusions from [145] and the previous subsection are correct. The contradiction is actually attributed to the difference of the targeted wave types to be harvested in the two different situations. In the previous subsection, the vibration-based energy harvesters are considered to be under base excitations. The vibration modes of the whole system, e.g. beam and plate, are intrinsically elastic standing waves superposed by an infinite number of travelling waves. In the band gap, the vibration of the whole system is suppressed due to the self-cancellation effect of the infinite number of travelling waves. Near system resonances, the vibration of the whole system becomes significantly intensive due to the constructive effect of the infinite number of travelling waves. For this reason, the maximum energy can only be harvested near the system resonant frequencies and very small amount of energy can be produced in the band gap.

The situation in [145] we are discussing for acoustic energy harvesting is different. The closed end of the impedance tube is placed with sound-absorbing foam to obviate sound reflection, playing the role as a perfectly matched layer. Thus, the incident acoustic waves in the ambient environment are travelling waves. When the acoustic wave frequency in the impedance tube matches the resonant frequency of the Helmholtz resonator, the sound pressure in the Helmholtz resonator is significantly amplified. Due to the reaction of the Helmholtz resonator, most of the incident wave is reflected back, and less is transmitted. The acoustic energy is concentrated at the reflecting interface and squeezed into the Helmholtz resonator. This explains why for the system presented in [145], using Helmholtz resonator can achieve peaks simultaneously in both sound transmission loss and power output responses.

To achieve broadband ability, instead of coupling a Helmholtz resonator with a membrane, Liu *et al* [30] directly introduced couplings between the Helmholtz resonators, as shown in figure 14. The rigid wall between the neighboring Helmholtz resonators was replaced by a flexible plate bonded with a piezoelectric transducer for energy transduction. The experimental results showed that over a wide range of frequency

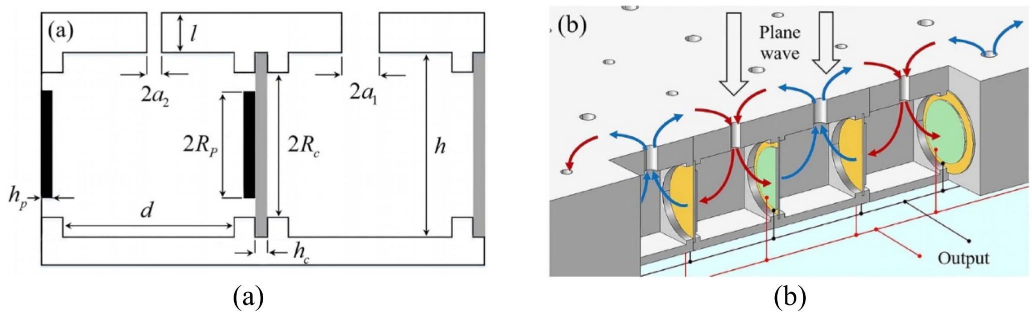

Figure 14. (a) Details of two coupled Helmholtz resonators, (b) the acoustic metasurface proposed in [30]. Reprinted from [30], with the permission of AIP Publishing.

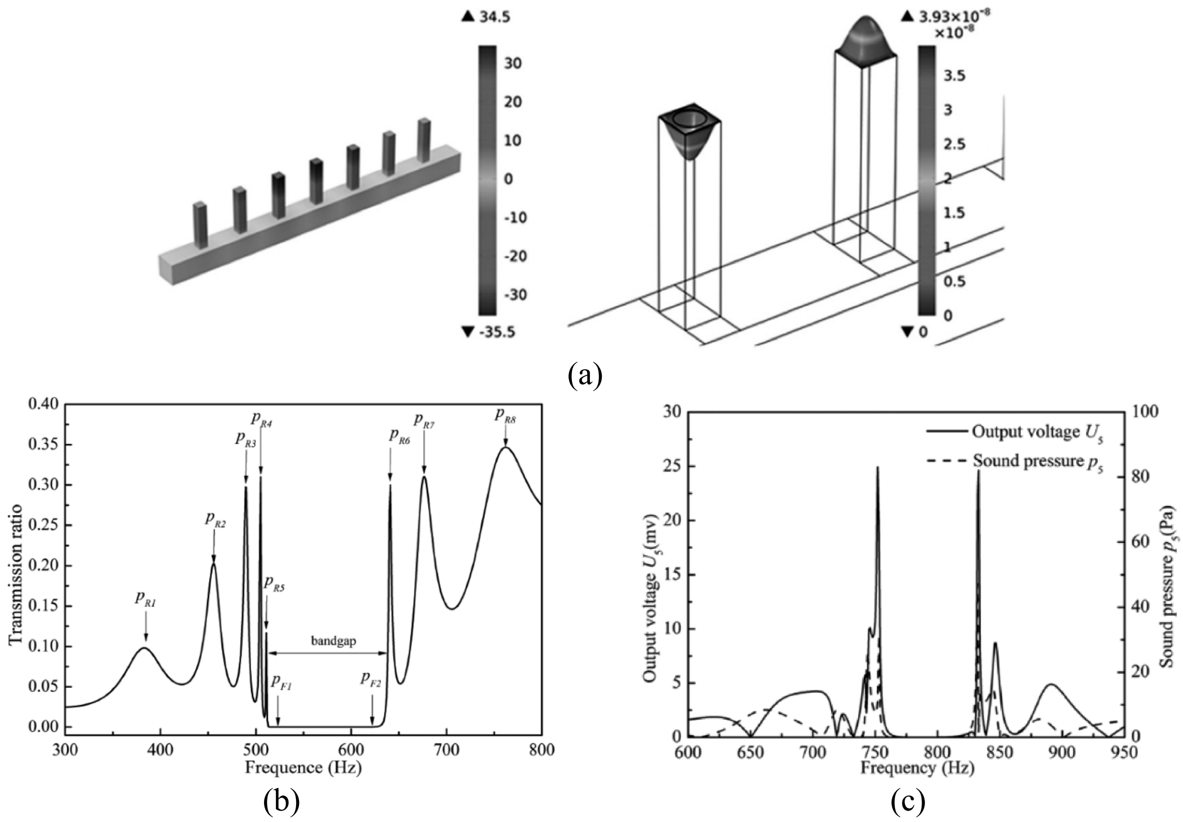

Figure 15. (a) Sound pressure distribution in the quarter-wavelength resonator based acoustic metamaterial and the elastic deformation of the piezoelectric transducer, (b) transmission spectrum, (c) the voltage frequency response [146]. Reproduced from [146].

from 460 to 680 Hz, substantially high voltage outputs can be obtained.

In addition to the Helmholtz resonator, the quarter-wavelength resonator is also widely used in the acoustic system design. Guo *et al* [146] proposed a quarter-wavelength resonator-based metamaterial system for energy harvesting, as shown in figure 15(a). The closed end of the quarter-wavelength resonator was replaced by a piezoelectric transducer to harvest the acoustic energy. A theoretical model was developed for predicting the sound transmission of the proposed acoustic metamaterial. The results shown in figure 15(b) verified the sound insulation capability of the proposed metamaterial. Through a finite element study, they also evaluated the voltage response (figure 15(c)) from the embedded piezoelectric transducer. By comparing figures 15(b) and (c), it is noted that the maximum voltage outputs are achieved at the system resonant frequencies, which are outside of the band gap. Though this work looks similar to that of [145] apart from the acoustic resonators being used, the conclusion is completely different: maximum sound isolation and energy harvesting can not be achieved simultaneously any more in this case. The subtle but crucial difference is that standing waves are formed in the quarter-wavelength resonator-based acoustic metamaterial proposed in [146]. The resultant acoustic system is similar to a mechanical metamaterial rod containing mass-spring local resonators [143]. Therefore, for such kind of system, as aforementioned in the previous subsection that the band gap can not entrap the energy. The optimal

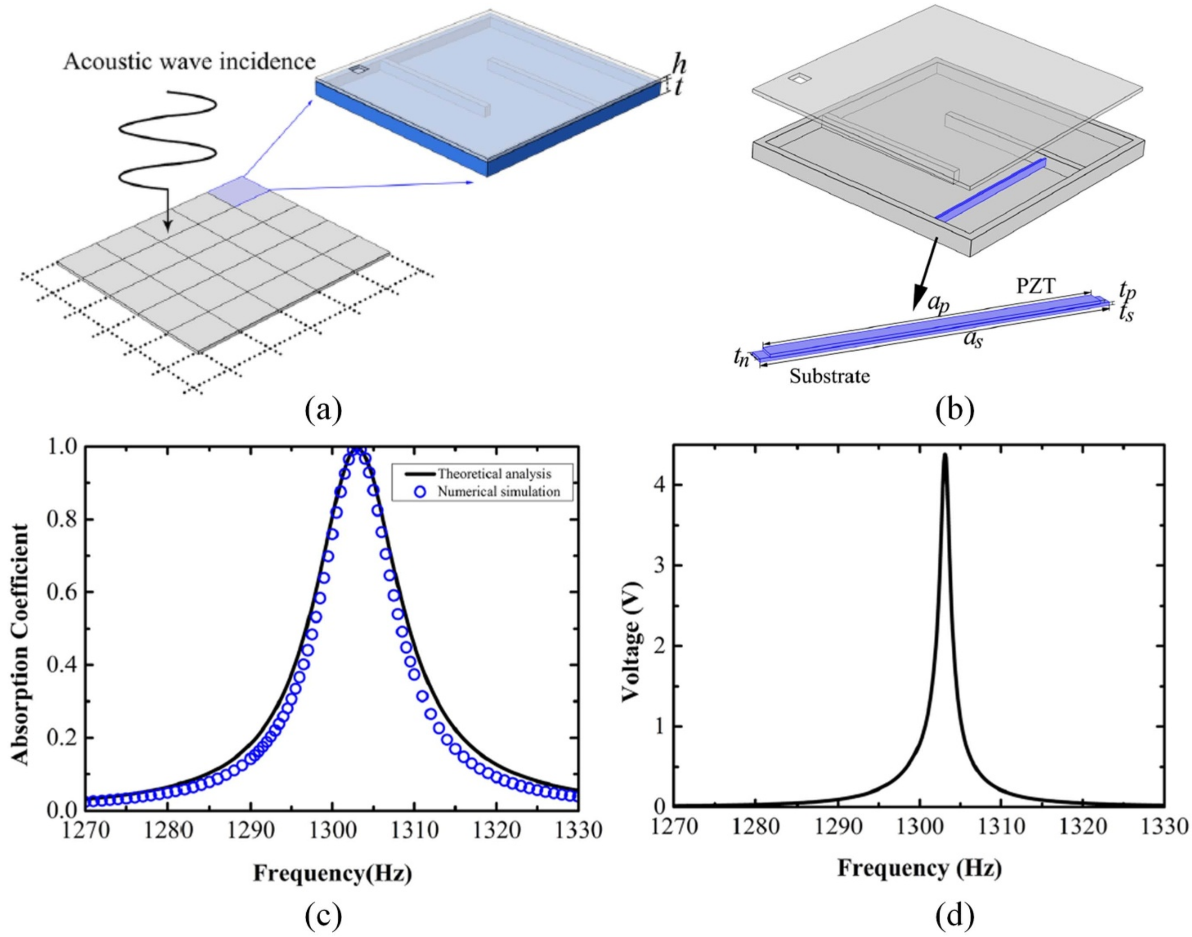

Figure 16. (a) The metasurface consists of labyrinthine resonators (b) the labyrinthine resonator embedded with a piezoelectric transducer for energy harvesting, (c) the absorption coefficient of the metasurface, (d) output voltage response of the piezoelectric transducer embedded inside the metasurface [147]. Reproduced from [147]. CC BY 4.0.

sound insulation/vibration suppression and energy harvesting can not be simultaneously achieved in the same frequency band.

Jin *et al* [147] developed an ultrathin metasurface (figure 16(a)) using labyrinthine resonators (figure 16(b)). The thickness of the developed metasurface is at deep sub-wavelength dimension. A piezoelectric transducer is inserted inside the labyrinthine resonator for harvesting energy. The mechanism of the labyrinthine resonator is actually similar to that of a traditional Helmholtz resonator that has a resonant frequency. Around the resonant frequency of the labyrinthine resonator, the metasurface exhibits the extraordinarily sound insulation ability which is revealed from the absorption coefficient curve, as shown in figure 16(c). On the other hand, from figure 16(d), it is noted that near the resonant frequency of the labyrinthine resonator, the inserted piezoelectric transducer can produce the maximum voltage output. The advantages of this metasurface include the ultrathin thickness as compared to the Helmholtz resonator type metamaterial and the high mechanical robustness as compared to the membrane type metasurface. Also, based on labyrinthine resonators, Sun *et al* [148] designed an acoustic metamaterial with the topological architecture similar to that presented in [144] for energy harvesting. Both numerical and experimental studies showed the improvement of the energy harvesting performance.

Topological metamaterial is a relatively undeveloped area in the community of metamaterials. Topological metamaterial is an analogy to the topological insulator in condensed matter physics [150]. The localization effect at the topological interface may be beneficial for improving energy harvesting efficiency [151]. However, using topological metamaterials for energy harvesting has been rarely reported. Fan *et al* [149] employed the interface localization state of topological metamaterial and proposed an acoustic energy harvester, as shown in figure 17. They first analysed the band-edge eigenmode of the studied topological metamaterial and verified the acoustic localization effect at the interface. Subsequently, they installed a piezoelectric transducer at the interface and evaluated the acoustic pressure using COMSOL simulation. It was found that the acoustic pressure at the interface was amplified by 66 times at the eigen-frequency of the interface mode (i.e. 4045 Hz). The voltage output from the piezoelectric transducer at this frequency was about 163 mV. The research in this direction is worth further investigation and more attention from researchers. For example, how to achieve the interface mode at low-frequency regime for low-frequency energy harvesting?

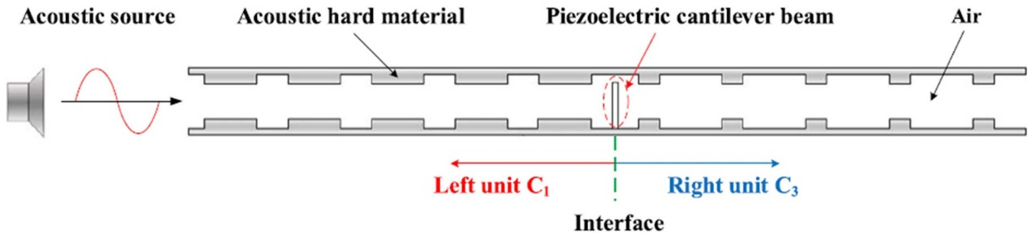

Figure 17. Schematic of a topological metamaterial based acoustic energy harvester [149].

A minor derivation from the eigen-frequency of the interface mode will result in a significant reduction in the output voltage amplitude. Similar to the idea of multi-mode energy harvesting, is there any means to produce multiple interface modes for energy harvesting? In addition, it is worth mentioning that the idea of generating the interface mode is similar to the idea presented in [152] of customizing vibration mode shapes. Though they are realized through different mechanisms, the purpose is the same, that is, to achieve a mode concentration effect for localizing dynamic energy through changing the mode shape.

According to the literature review, one can find that, for most of the existing multifunctional metamaterial/PC energy harvesting systems, the vibration suppression function and the energy harvesting function can only work in separate frequency ranges. Hence, vibration suppression and energy harvesting can not be literally achieved simultaneously by most of current multifunctional metamaterial/PC based energy harvesters under narrow-band excitations. Some statements in the literature, such as that 'the energy is absorbed by the local resonators, thus the maximum energy can be harvested in the band gap of metamaterials' are actually confusing in some sense, since the working condition is not well explained. In fact, within the band gap, the whole system is at a state similar to anti-resonance. Though the mechanical energy density in the local resonators are comparatively much larger than that in the main structure whose vibration is targeted to be suppressed. The total vibration energy transferred to the entire structure is very small in the band gap as compared to the resonance state of the system [115]. The results of the study in [115] clearly showed that the optimum frequency ranges for vibration suppression and energy harvesters are non-overlapped, and the conditions for the optimal vibration suppression and energy harvesting are different. Therefore, we can conclude that the band gap phenomenon does not have any benefits for energy harvesting using standing wave systems. The results in [145] showed that in a travelling wave system, due to the band gap phenomenon, incident waves can be reflected back and concentrate in the local resonator at the reflection interface. In this situation, maximum sound isolation and energy harvesting can be truly achieved simultaneously.

Overall speaking, most multi-functional metamaterial-based energy harvesters in the literature are developed on the basis of easy implementation of metamaterial structures. The energy harvesting function is more like a by-product that can be conveniently obtained by introducing an electromechanical transducer into a readily existing metamaterial structure. From this point of view, a multi-functional configuration is usually not meant to bring too many improvements for energy harvesting. But from another point of view, this is also the advantage of directly integrated multi-functional configurations. Researchers can devote most of their efforts to the development of metamaterial systems and then obtain the bonus energy harvesting function in an economical way.

In retrospect to the literature reviewed in this section, one may notice another fact that almost all the existing work are based on using linear metamaterials for energy harvesting. On the one hand, in the community of energy harvesting, nonlinear oscillators have been extensively explored for energy harvesting [65, 66, 153–156]. It has been revealed that the bending phenomenon of the amplitude-frequency response curve around resonance due to the nonlinearity results in a broader range of frequency for energy harvesting. On the other hand, introducing nonlinearity is a strategy to achieve wide band gaps for metamaterials [157–159]. In particular, Fang *et al* [158] explained that ultra-broad band gaps can be obtained in nonlinear metamaterials for the induced chaotic motions. However, using nonlinear metamaterials for energy harvesting is still very rare. In recent, Hwang and Arrieta [160] conducted a preliminary study of a metamaterial that is composed of bistable unit cells for energy harvesting. This study represents an early attempt to investigate nonlinear metamaterials for energy harvesting. However, the analysis is insufficient to reveal the in-depth mechanism and phenomenon that can benefit energy harvesting efficiency. Researchers from the communities of nonlinear metamaterials and nonlinear energy harvesting may pay more attention to this direction to promote its development.

4. Defect state based configurations

The band gap is a unique phenomenon of metamaterials and PCs that has great engineering values for vibration suppression or noise reduction. However, from the results of the work summarized in the previous section, we can conclude that the band gap phenomenon itself does not have any benefit for improving energy harvesting efficiency. Defect state is another interesting phenomenon in metamaterials and PCs. A defect metamaterial/PC is obtained by breaking the periodicity at a certain place. A defect mode will then appear at a specific frequency within the band gap. To predict the appearance of defect mode frequency, one can use the conventional dispersion relation calculation methods based on an assumed

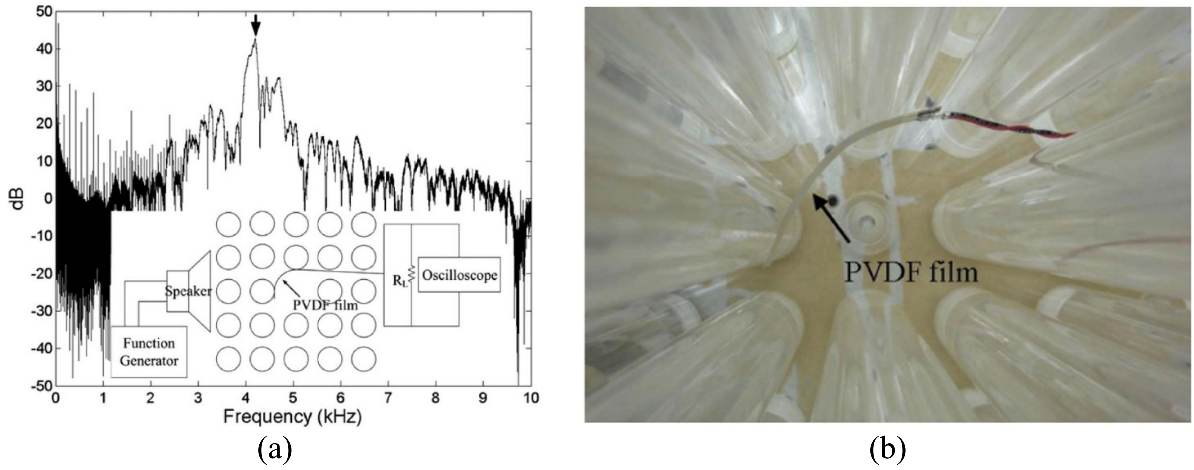

Figure 18. (a) Schematic of a defect sonic crystal for energy harvesting [163], (b) physical implementation of placing a PVDF film inside of the cavity of sonic crystal for energy harvesting [29]. Reprinted from [163], with the permission of AIP Publishing.

super-cell [161, 162]. For the defect mode, it has been proved that the wave of the defect mode frequency can be localized at the defect place of the metamaterial/PC [162]. For this reason, researchers have been attracted to employ the energy localization effect at the defect mode for improving the energy harvesting efficiency. Based on the wave properties, we can also classify the related research in this section into acoustic and elastic types. However, different from section 3, the subsection arrangement of this section is determined based on the following classification criterion for more easily indicating the installation features of the systems. According to the relationship between the incident wave direction and the characteristic plane of metamaterial/PC, the defect state based configuration can be further divided into two groups, namely, the in-plane mode and the out-of-plane mode.

4.1. In-plane mode

Wu *et al* [163] developed a defect sonic crystal for energy harvesting, as shown in figure 18(a). It is worth mentioning that 'sonic crystal' is an alternative name of PC to refer to acoustic PC in particular. Due to the introduction of the defect, a band corresponding to the defect mode would appear in the band structure. The defect mode of the developed sonic crystal was confirmed to appear at 4.21 kHz, which is because the lattice constant of the studied sonic crystal is very small (only 49 mm). From the experiment, it was found that the pressure in the defect cavity was enhanced by 4.94 times and power by 24.4 times as compared to the sonic crystal without defect. In a later study by the same research group [29], they presented the physical prototype as shown in figure 18(b). A model of the sonic crystal and a model of the piezoelectric energy harvester have been developed. The theoretical model was validated by the experimental result. However, due to the low power density in the acoustic waves, the harvested energy power reported in [29, 163] is limited at nanowatt level.

It is well known that a Helmholtz resonator has the ability to amplify sound pressure. Yang *et al* [164] installed a Helmholtz resonator in the defect cavity of a sonic crystal to further enhance the energy harvesting performance. The structure of the entire system is illustrated in figure 19(a). The frequency range for energy harvesting is around 5 kHz and the power output is increased to the microwatt level. The introduction of the Helmholtz resonator resulted in the coupling with the defect mode. The experimental results showed that due to the resonance coupling, the energy harvesting efficiency was increased by 23 or 262 times as compared to the case when a defect sonic crystal or a Helmholtz resonator was employed, separately. Later on, researchers from the same group proposed to use two coupled defect sonic crystals for further enhancing the acoustic wave localization effect [165]. Though they did not directly use it for energy harvesting, from their experimental results, we can expect that the coupled system of the two defect sonic crystals should have the potential for improving energy harvesting efficiency as well. The schematic of the proposed system is presented in figure 19(b). This idea is naturally inspired from that of the Helmholtz resonator coupled defect sonic crystal analysed in their previous study [164]. They first calculated the defect modes of the two different defect sonic crystals independently. The one with a smaller lattice constant produced a defect band in the band structure at a higher frequency range. When the two defect sonic crystals are coupled, there exist two defect bands. They measured the pressure in the experiment and validated the pressure amplification effect around the eigen-frequencies of the two defect bands.

In the study of [164] and [165], Yang *et al* used cylinder inclusions to design the sonic crystals. In a following study of [166], they proposed to use cross-plate inclusions to increase the Q-factor of the sonic crystal. The Helmholtz resonator is still used to form coupling with the defect sonic crystal. The entire structure of the system is similar to that presented in figure 19(a), except the cylinders are replaced by cross-plates. Using the high Q sonic crystal, they claimed that the output power from the energy harvester was further improved by 22 times than that reported in [164] with cylinder inclusions.

Based on the same idea of using defect mode, figure 20(a) presents a defect metamaterial designed by Carrara *et al* [167]

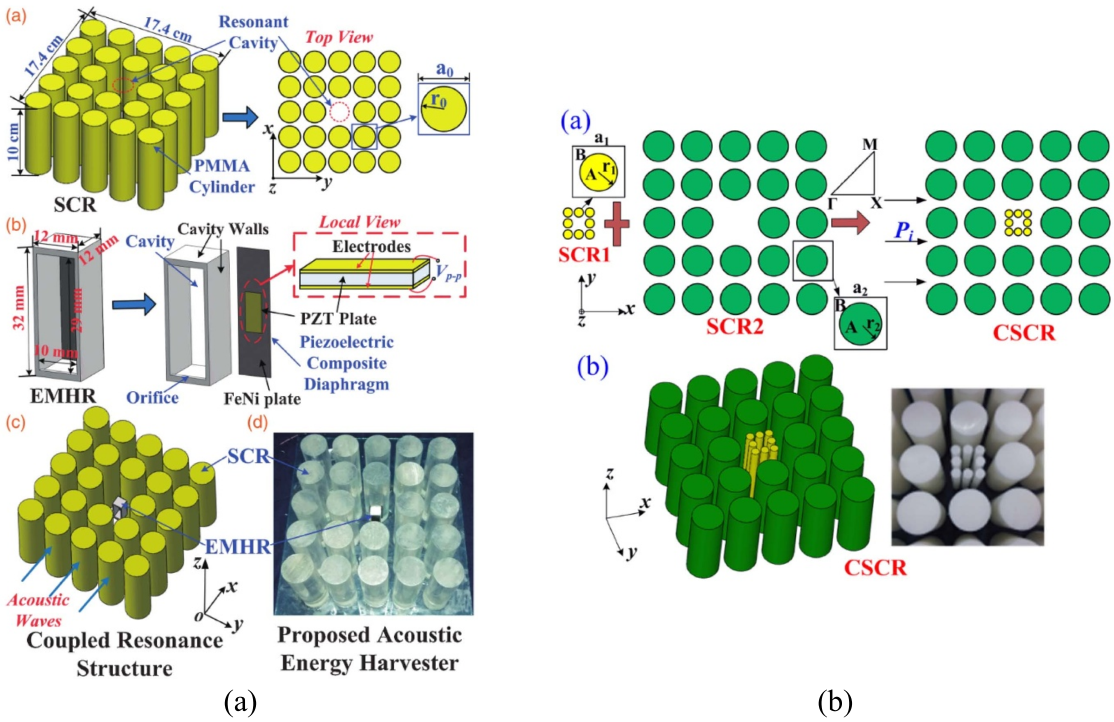

Figure 19. (a) A piezoelectric transducer coupled Helmholtz resonator being installed in a defect sonic crystal to achieve enhanced energy harvesting performance [164]. (b) Schematic of two coupled defect sonic crystals [165]. Reprinted from [165], with the permission of AIP Publishing.

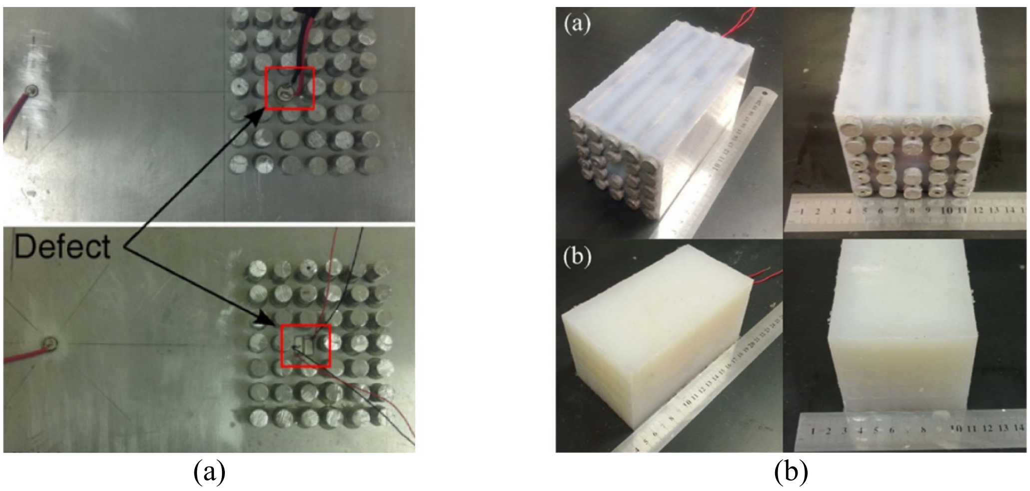

Figure 20. (a) PC with point defect. The above figure shows the configuration with a circular piezoelectric energy harvester, the bottom figure shows the configuration with two rectangular piezoelectric energy harvesters [167]. (b) A vibration energy harvester using the point defect state of a PC. The above figure shows the defect PC and the bottom figure shows a rubber block for comparison [168]. Reprinted from [168], with the permission of AIP Publishing.

for harvesting energy from elastic waves. The experimental result showed that two defect modes at frequencies of 35 and 63 kHz were obtained for energy harvesting. Though the defect mode frequency is also very high, at the same level as the acoustic case [163], the experimental result showed that the harvested energy power was at microwatt level which is much larger than that of the acoustic counterpart [163]. Lv *et al* [168] also designed a similar defect PC for vibration energy harvesting, as shown in figure 20(b). For their designed defect PC, the eigen-frequency of the defect mode is around several hundreds of Hertz, which is much lower than the cases presented in [163, 167, 169]. They also fabricated a pure rubber block without the defect mode for comparison. At the eigen-frequency of the defect mode, the power output from the energy harvester being placed in the defect PC was found to be 177 421 times larger than that being placed in the rubber block without the defect mode.

For a similar defect sonic crystal based energy harvester as presented in figure 18(a), Aly *et al* [170] explored the temperature effect on the energy harvesting performance using

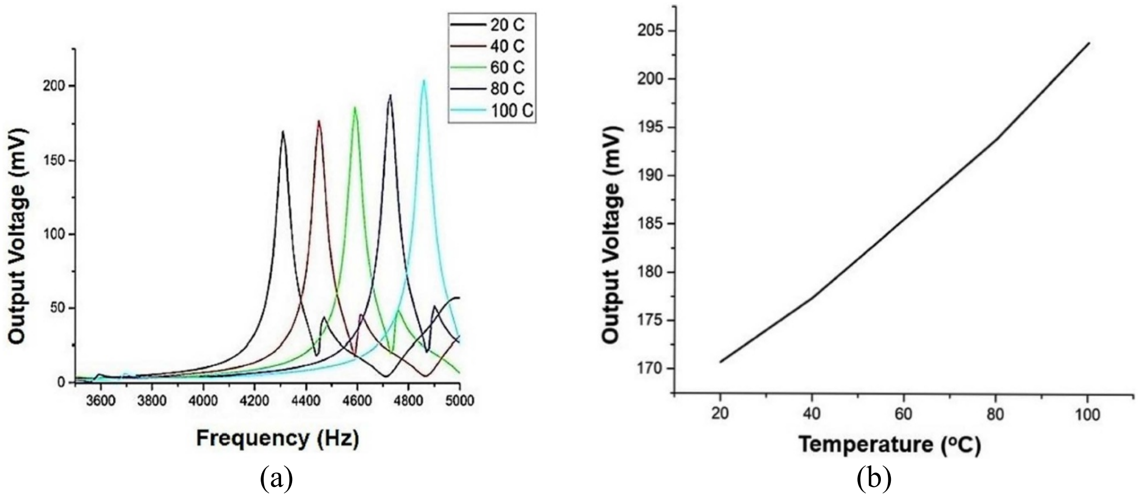

Figure 21. (a) Voltage frequency responses at different temperatures, (b) output voltage versus temperature [170]. Reprinted from [170], with the permission of AIP Publishing.

COMSOL simulation. From their results shown in figure 21, it is noted that with the increase of the temperature, the output voltage increases. Moreover, the resonant frequency shifts to the higher frequency range. They explained that this is due to the increase of the piezoelectric constant with the increase of the temperature. In another work [171] by Aly *et al* as well, the temperature effect on the pass band of a one-dimensional PC that contained a piezoelectric material constituted defect was theoretically investigated. Though the environmental temperature must influence practical materials/devices, research of the thermal effect on piezoelectric metamaterial systems is still rare. Researchers could possibly devote more efforts in this direction in the future.

Instead of adding inclusions onto plates to achieve a PC, Jo *et al* [172] introduced some holes in a plate and obtained a PC plate, as shown in figure 22(a). An in-plane excitation incident from one side of the PC plate was considered. A piezoelectric transducer was introduced in the defect cell for energy harvesting. The previous studies introduced only a single defect in the PCs for energy harvesting. Jo *et al* proposed to introduce two coupled defects to split the defect band for widening the frequency bandwidth for energy harvesting. As expected, from the voltage frequency responses obtained from simulation shown in figure 22(b), two defect resonant frequencies appeared where the output voltage was substantially large. Aslo through finite element simulation, they inspected the two defect modes formed by the coupling of the two defects and verified their speculation. Though the lattice constant is 33 mm, which is at the same scale of those in [163, 164, 168, 173], the two defect modes appeared at very high frequencies: 59.79 and 60.11 kHz, respectively. Using a similar structure, the same research group has also conducted some related work which has been reported in [174]. Another work from them provided a comprehensive experimental study for the validation [175]. The experimental results showed that the output power from the piezoelectric energy harvester was increased up by 22.8 times as compared to the case without using the defect PC.

Based on a beam structure, Hu *et al* [176] developed a phononic beam-based piezoelectric energy harvester which is similar to the system presented in [118]. The difference is that a cavity defect is introduced in the phononic beam and a single piezoelectric transducer is used and installed in the cavity. The theoretical analysis predicted the occurrence of the defect mode band in the band structure. The COMSOL simulation was conducted to predict the power output from the energy harvester. Also, for a phononic beam consists of binary beams, as illustrated in figure 23(a), Geng *et al* [177] investigated the potential of its defect state for energy harvesting through theoretical analysis. Interestingly, they considered the model in the thermal environments and explored the effect of the temperature on the band structure behaviour and the energy harvesting performance. It is found that with the increasing temperature, due to the softening effect of the thermal load on the beam, the energy harvesting performance is depressed. For instance, when the temperature was increased by 60 °C, the output voltage was reduced by more than three times, as indicated in figure 23(b). It is worth noting that this conclusion is different from that given in [170]. This is because they considered the thermal effect on the beam instead of the piezoelectric material. The thermal effect on the beam is incorporated in the governing equation of the beam as a thermal induced axial force. If both the thermal effects on the beam and the piezoelectric constants of the piezoelectric material are considered, the results could be different. Further efforts may be devoted to investigating this problem, especially through some experimental studies for validation. On the other hand, with the increase of the temperature, the band gap, together with the defect mode band, moves to a lower frequency range (as demonstrated in figure 23(c)), which is beneficial for low-frequency vibration suppression and energy harvesting.

4.2. Out-of-plane mode

Different from the studies reviewed in section 4.1, Qi *et al* [28] presented a defect sonic crystal, as shown in figure 24,

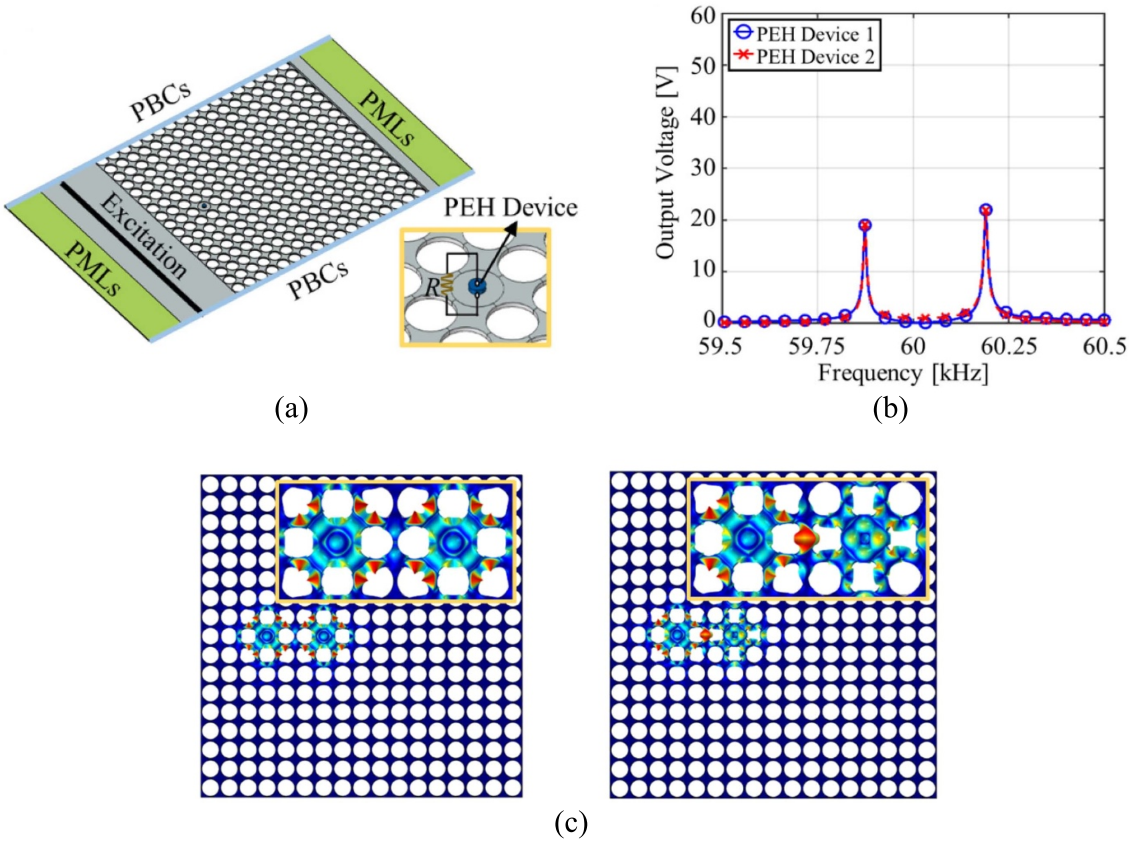

Figure 22. (a) Defect PC plate with piezoelectric transducer, (b) open-circuit voltage frequency response, (c) two defect mode shapes: in-phase mode (left), out-of-phase mode (right) [172]. Reprinted from [172], with the permission of AIP Publishing.

for harvesting energy from the acoustic waves in the direction in parallel with the cylinder inclusions. Another difference is that the cylinders are installed on a thin elastic plate. When the sound is incident, the acoustic-structure coupling takes place. According to the dispersion relation study using supercell calculation method, a defect mode is formed at the frequency of 2257.5 Hz due to the introduction of the defect. Then, they developed a finite element model using COMSOL and performed simulations to evaluate the voltage outputs from the piezoelectric transducer around the eigen-frequency of the defect mode. Numerical results showed that for an incident acoustic pressure of 2 Pa, the obtained power density is around 0.54 μW cm^{-3} at the frequency of 2257.5 Hz.

To realize the tunability of the defect mode for energy harvesting, Deng *et al* [178] proposed a magneto-elastic planar sonic crystal, as shown in figure 25(a). The cylinder pillars are made of magnetostrictive materials. The mechanical modulus of the magnetostrictive material is dependent on the magnetic field. Thus, by varying the applied magnetic field, the band structure of the proposed metamaterial can be tuned. Figure 25(b) shows the band structure results under different magnetic fields simulated by COMSOL. It can be expected that the voltage output from the embedded piezoelectric transducer will change for different magnetic fields. The corresponding simulated results in terms of the voltage output is demonstrated in figure 25(c). Another research from the same research group presents some further studies of the same magneto-elastic sonic crystal for energy harvesting [179]. In the modelling of the proposed magneto-elastic planar sonic crystal, the nonlinear magneto-mechanical coupling was linearized for simplicity. In the analysis, in addition to the out-of-plane modes, they also discussed the in-plane modes.

The previous work is all based on creating defects in PCs; for locally resonant metamaterial, Oudich and Li [173] proposed some defect configurations for energy harvesting, as shown in figure 26. To form the defect, they explored the ways of introducing an empty cavity and changing the mechanical properties of one or several local resonators. The benefit of the later configuration is that the eigen-frequency of the defect mode can be tuned by changing the mechanical properties of the local resonators. They developed an analytical model to analyse the band structure, sound transmission loss, and the energy conversion of the proposed defect metamaterial plate. Moreover, finite element simulations were performed for evaluating the energy harvesting performance. The eigen-frequencies of the defect mode are around 500 Hz. The collected power is about 22 μW.

Compared with the multifunctional configurations reviewed in section 3, defect state based configurations exhibit remarkable enhancements of energy harvesting performance: the power output can often be improved for several dozens or even hundreds of times. However, there are several drawbacks of defect state based configurations. First, most of defect state based configurations are realized using PCs; the

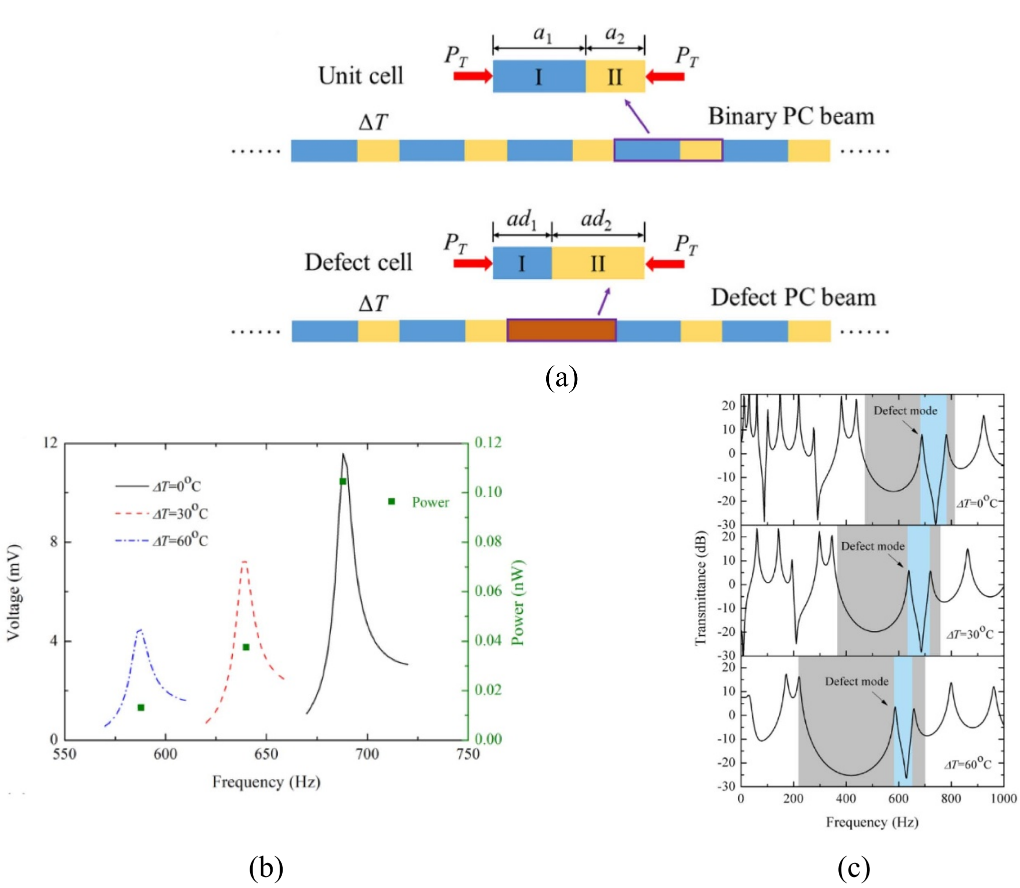

Figure 23. (a) Schematic of a perfect and a defect phononic beam in the thermal field, (b) open-circuit voltage responses for different temperature rises, (b) transmittance response for different temperature rises [177]. Reprinted from [177], with the permission of AIP Publishing.

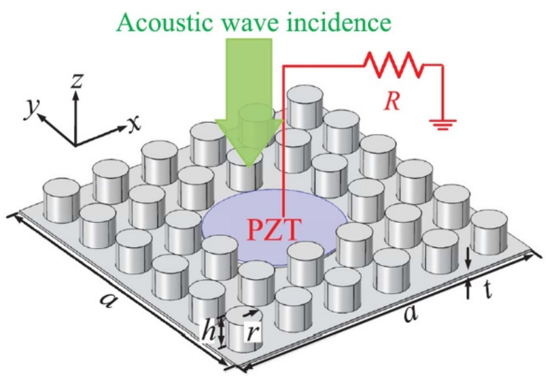

Figure 24. A defect sonic crystal embedded with a piezoelectric transducer for harvesting energy from acoustic waves that is incident in the direction in parallel with the cylinder inclusions [28]. Reprinted from [28], with the permission of AIP Publishing.

operating frequencies are thus unavoidably located in the high-frequency range at the level of kHz. As most ambient vibration energy spreads over the low-frequency spectrum [3], this high-frequency characteristic of defect state based configurations goes against the urgent demand of low-frequency energy harvesting. Second, a defect state based configuration consists of a defect metamaterial/PC and a conventional energy harvester being installed in the defect cavity. The defect metamaterial/PC plays the role of an auxiliary device to realize energy amplification. In this manner, the dimension of the auxiliary device, i.e. the defected metamaterial/PC, is actually much larger than the core component, i.e. the energy harvester. Though the power output can be significantly amplified, the power density of the whole system including the relatively 'giant' auxiliary device is actually questionable. Last, the operation bandwidths of defect state based configurations are commonly very narrow. The operation band is originated from the defect state which is obtained by breaking the integrity of a metamaterial/PC at the sacrifice of a complete band gap. Almost all the existing studies of defect state based configurations focused on the investigation from the perspective to improve the energy harvesting performance, while ignored the influence of the defect for other application scenarios, such as vibration suppression or noise reduction. Therefore, the engineering value of the defect state based configurations is still doubtable and needs to be validated through application in practical scenarios.

5. Wave focusing mechanism enhanced configurations

From the existing work summarized in sections 3 and 4, it can be noted that the operation bandwidths of both the directly

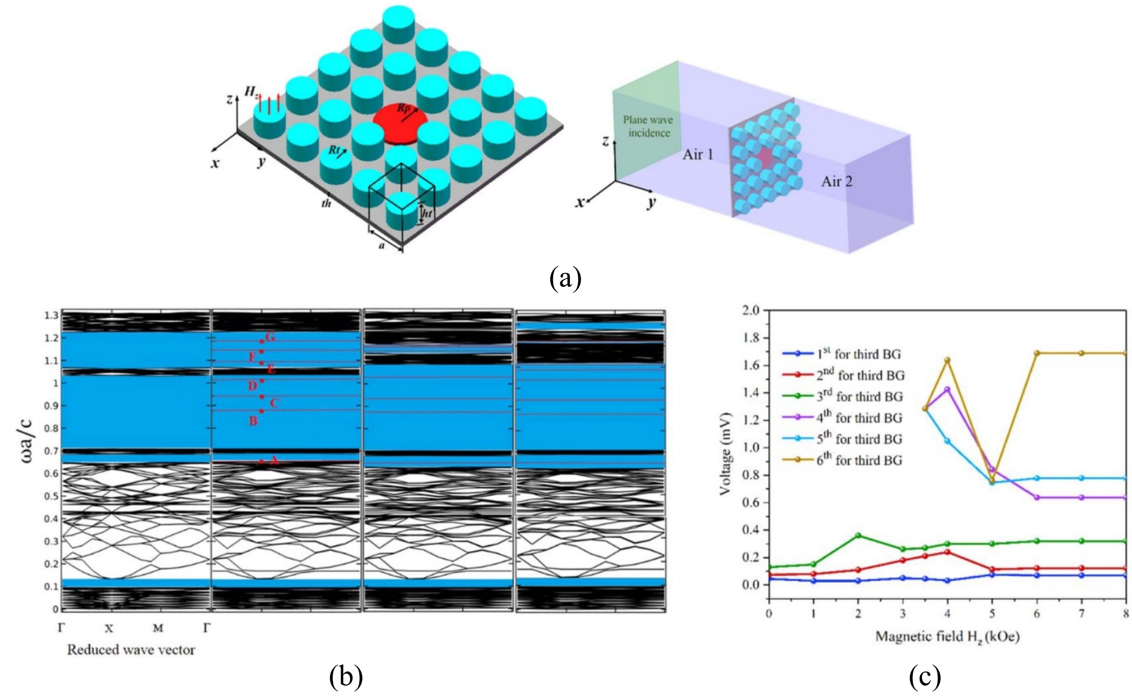

Figure 25. (a) Magneto-elastic planar sonic crystal, (b) band structures under different magnetic fields, (c) output voltage for the defect bands in the third band gap under different magnetic fields [178].

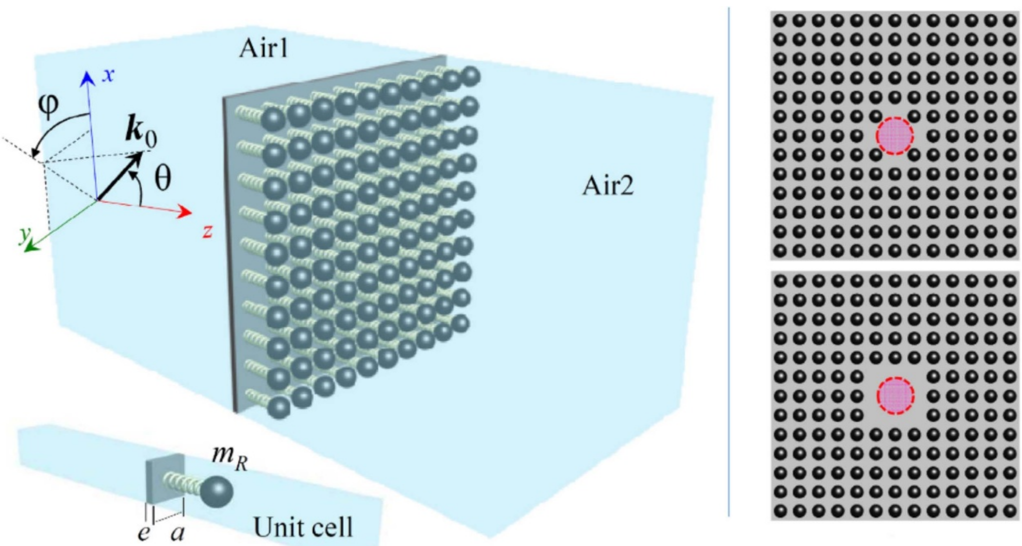

Figure 26. Schematic of the metamaterial plate in the airborne sound filed (left), the defect metamaterial configurations (right) [173].

integrated multifunctional configuration and the defect state based configuration are still limited. The multifunctional configuration is similar to the conventional multi-modal energy harvesters and can deliver substantial power output only around the structural resonance. The defect band of the defect state based configuration is very flat and narrow, thus in the power frequency response, there only appear several peaks at discrete frequencies where considerable energy can be harvested. For the above reasons, we can conclude that these two configurations can not benefit for broadband energy harvesting. The third kind of configuration to be introduced in this section overcomes this disadvantage by focusing waves over a wide frequency range. To achieve wave focusing, the PCs and metamaterials are designed to form the mirror/lens effect. The related research in this section could be classified into acoustic and elastic types as well. However, classifying them according to the formation of the mirror/lens effect is obviously more rational, making them more recongnizable from the underlying mechanism perspective.

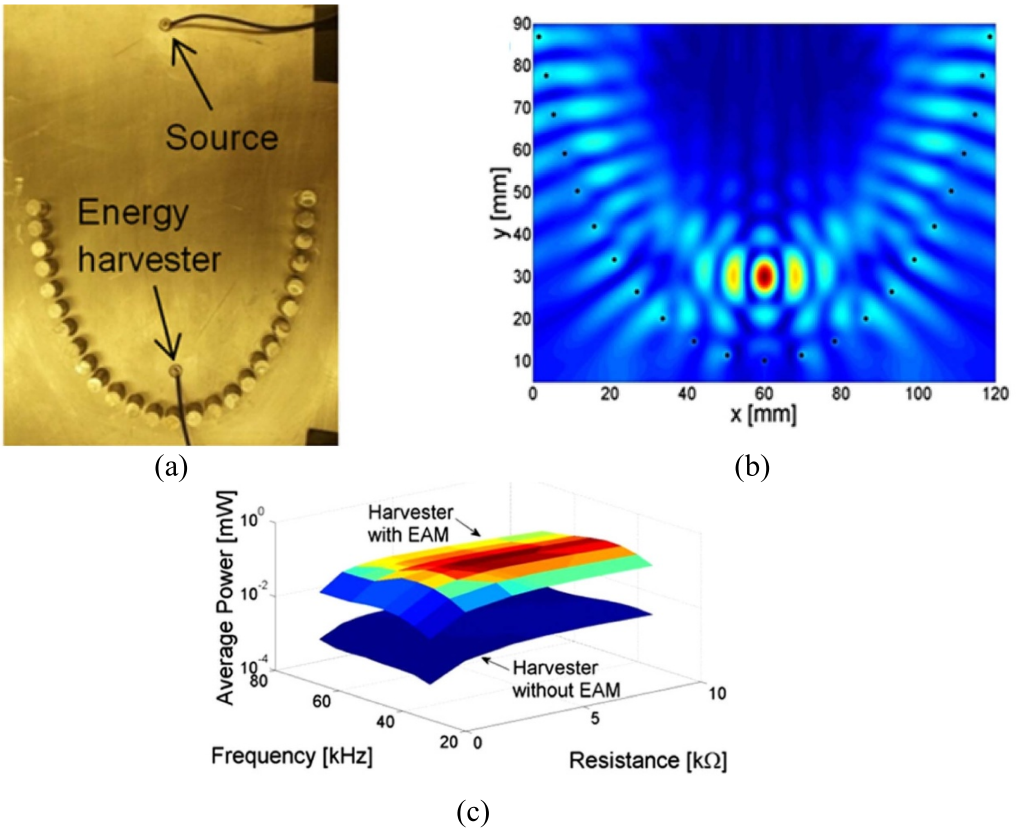

Figure 27. (a) PC stubs arranged in the elliptical shape to achieve acoustic mirror effect, (b) simulated wave focusing phenomenon due to the acoustic mirror effect, (c) comparison of the power output from the energy harvesters with and without the acoustic mirror effect [180]. Reprinted from [180], with the permission of AIP Publishing.

5.1. Mirror type

Carrara *et al* [180] exploited a PC inspired structure that is arranged in the elliptical shape for energy harvesting from structure-borne waves. Figure 27(a) shows the physical implementation of the whole setup, including the installation of the piezoelectric energy harvester. Such kind of arrangement will produce the acoustic mirror effect that can focus the elastic wave energy to a specific point. Figure 27(b) shows the simulated wave focusing phenomenon due to the acoustic mirror effect. The condition to achieve the acoustic mirror effect is to make the spacing between the cylinder inclusions smaller or of the same order of the wavelength of the considered Lamb wave mode. In their experiment, they tested the energy harvesting performance over the frequency range from 25 to 150 kHz. To directly evaluate the energy harvesting performance, figure 27(c) compares the power output from the installed PEHs with and without the acoustic mirror effect. It can be seen that because of the acoustic mirror effect, the power output from the installed PEH was increased by more than 1 order of magnitude. Therefore, the proposed PC structure can not only improve the power output of the PEH but also widen the operation bandwidth of the PEH. Based on the same idea of the acoustic mirror [180], Tol *et al* [181] redesigned the mirror structure by embedding beads into the host plate. Based on the COMSOL simulation, they analysed the wave focusing capability of the bead embedded acoustic mirror. A further study of using beads embedded PC to design acoustic mirrors for energy harvesting can be found in [182].

As an extension of the PC formed acoustic mirror presented in [180], the same research group proposed a PC formed acoustic funnel for energy harvesting [167]. Figure 28(a) shows the prototyped acoustic funnel, which is obtained by opening a funnel based on a structure similar to the acoustic mirror. The waves within the frequency ranges of the band gaps will be confined within the designed region and can only propagate into the desired funnel. A PEH is then installed in the channel for harvesting the guided wave energy. Figure 28(b) compares the power output from the PEHs installed inside and outside the funnel. It can be observed that the PEH installed inside the funnel produced a much larger power output.

For the PC formed acoustic mirror, Darabi and Leamy [183] developed an in-depth theoretical model using the multiple scattering formulation. The presence of the piezoelectric energy harvester was also included in the modelling to guarantee the completeness of the electromechanical coupled system. Based on the theoretical model, they conducted an optimization study towards improving the energy harvesting performance. The optimized system exhibited an enhanced output power by ten times. An experimental study validated their theoretical model. Overall speaking, their work presents a comprehensive and rigorous modelling of the PC formed

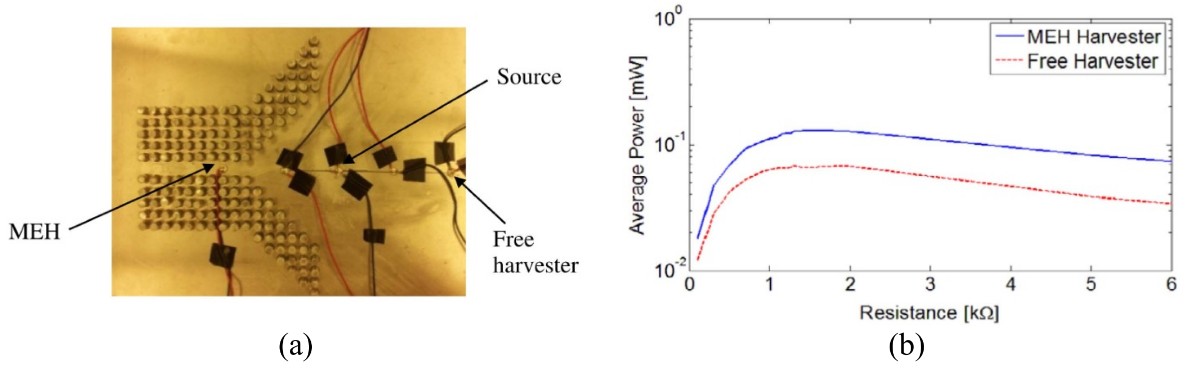

Figure 28. (a) PC formed acoustic funnel for energy harvesting, (b) comparison of the output power from the PEH installed in the PC formed acoustic funnel and a free PEH installed directly on the plate [167].

acoustic mirror, and thus has a significant value for guiding the design of suck kind of energy harvesters.

Based on the generalized Snell law in the field of optics, Qi and Assouar [184] tailored acoustic waves using some labyrinthine units to confine the sound energy at a desired position for benefiting energy harvesting. Figure 29(a) shows the designed labyrinthine unit and the phase shift phenomenon of the acoustic wave tailored by the labyrinthine unit. Using the labyrinthine unit, they further designed the metasurface as shown in figure 29(b). The labyrinthine units at different positions were designed with different geometric dimensions to meet the requirement of the desired phase shift profile for the formation of a focal spot at a specific position. A finite element simulation presented in figure 29(c) verified their conceptual design, in which besides the source point, one can observe a spot with the maximum pressure amplitude. By intentionally installing the piezoelectric energy harvester at the designed focal spot, the energy harvesting efficiency can be significantly improved and the results (as shown in figure 29(d)) obtained from their simulation well supported their prediction. However, for the given design, this metasurface can only confine acoustic waves at the frequency of 3430 Hz to the designed focal spot, which indicates that its working frequency for energy harvesting enhancement is only around 3430 Hz.

5.2. Lens type

In analogy to an optical lens, Tol *et al* [185, 186] designed a gradient-index (GRIN) PC lens to focus elastic Lamb waves for energy harvesting. The underlying mechanism is similar to that of an optical lens. By varying the inclusions of the PC, one can customize the refractive index along the direction transverse to the wave propagation direction to have a hyperbolic secant gradient distribution. When a plane wave is incident, the waves at both side of the center axis will be bent to the center axis at a focal spot. Figure 30(a) illustrates the wave trajectory in the GRIN-PC to help understand this mechanism. Figure 30(b) shows the fabricated physical prototype and the experimental setup. Then, in the experiment, the authors measured the RMS velocity field under the excitation of different frequencies. At a specific position different from the source position, a maximum intensity could be observed in the RMS velocity field, which indicated formation of the focal spot. The wave focusing effect could take place over a wide frequency range from 30 to 70 kHz. To evaluate the wave focusing effect on the energy harvesting performance, they compared the voltage and power outputs from the GRIN-PC lens-based PEH and the baseline PEH (i.e. without using the GRIN-PC lens). From figure 30(c), we can note that by using the GRIN-PC lens for focusing the wave, the energy harvesting efficiency was increased by 13.8 times compared with the baseline case when there is no GRIN-PC lens. The researchers from the same group then investigated the feasibility of employing 3D printing technique for fabricating the proposed GRIN-PC lens and tested its performance for energy harvesting [187, 188]. However, due to some reasons such as the effect of the thin epoxy bonding layer on the refractive index distribution, the wave focusing capability of the 3D printed GRIN-PC lens was not as good as expected. A similar design has also been employed by Hyun *et al* [189] for harvesting energy from flexural elastic waves.

Without using discrete inclusions, Zareei *et al* [190] designed a GRIN lens by directly varying the thickness of the plate to tune its rigidity for exhibiting the desired refractive index profile. Figure 31 shows the corresponding design principle. One benefit of this design is that the plate thickness can be varied continuously; thus, the refractive index profile will be very smooth, which can not be achieved using discrete inclusions. Though the structure design is different, the obtained wave focusing phenomenon is the same. The experimental results validated the occurrence of the focal spot. The improvement of the energy harvesting efficiency was successfully achieved as expected.

To achieve the desired refractive index profile, instead of varying the mechanical parameters of the inclusions, Yi *et al* [191] proposed to use piezoelectric patches to realize the inclusions, then shunt them to negative capacitances for controlling their effective Young's Modulus (figure 32). Using the negative capacitance to achieve tunable metamaterials was actually proposed by [39, 40]. More detailed information on this perspective can be found in these two references. Except for the difference in the methods for tuning the effective mechanical property, the design strategy of the PC is still the same. In the modelling of the piezoelectric energy harvester,

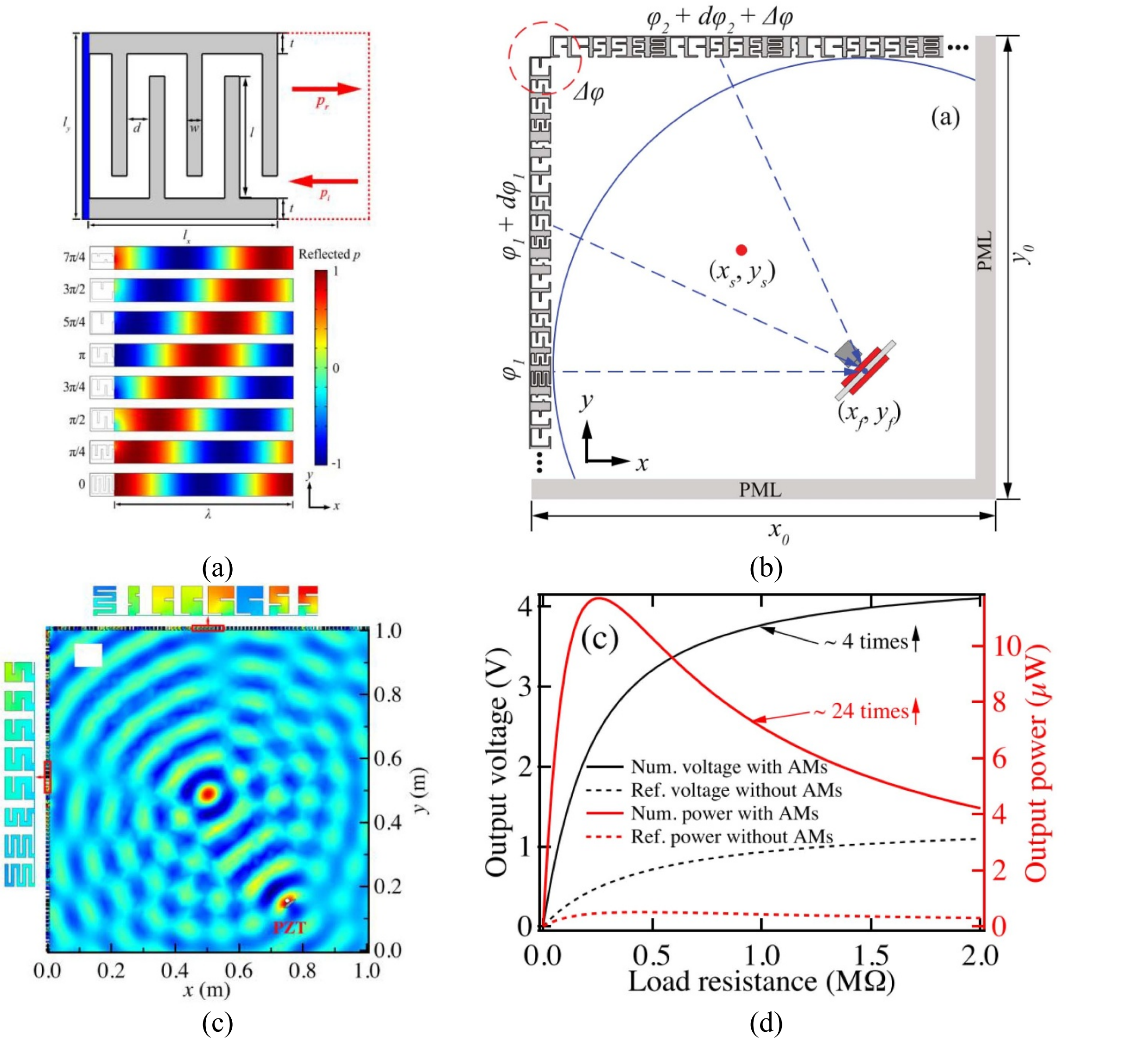

Figure 29. (a) Schematic of a labyrinthine unit and the phase shift caused by the labyrinthine unit with different geometric dimensions, (b) a partially enclosed region by the metasurface consisted of the labyrinthine units and a piezoelectric energy harvester used for energy harvesting, (c) the sound pressure distribution for the presence of the metasurface, (d) the voltage and power outputs from the piezoelectric energy harvester under the conditions when the metasurface is and is not used [184]. Reprinted from [184], with the permission of AIP Publishing.

the authors considered the effects of two advanced interface circuits (i.e. P-SSHI and S-SSHI) on the energy harvesting. According to the simulation results, it is revealed that using the advanced interface circuits can boost the efficiency of the piezoelectric energy harvester in the proposed PC lens.

Though the PC formed lens can be used for energy harvesting, the working frequency range is actually relatively high and out of the typical ambient energy spectrum. To reduce the working frequency, Tol *et al* [192] also proposed a locally resonant metamaterial-based lens for energy harvesting. The design of a metamaterial-based lens is similar to that of a PC based counterpart. Their proposed mechanical architecture is also very similar. The difference is that they used relatively soft acrylic plastic pillars and attached tip masses to the pillars. The pillar, together with a tip mass, served as a local resonator. Using such kind of strategy and following the same design procedure, they designed a metamaterial-based lens with the working frequency range being reduced to several hundreds of Hertz.

The aforementioned GRIN PC lens was designed in a rectangular shape and can only focus a unidirectional wave [185]. To realize the focus of omnidirectional waves, inspired by the circular shape of a Luneburg lens in the optics field, the researchers from the same group then developed a PC formed Luneburg lens and applied it for energy harvesting [193]. The underlying mechanism is actually similar to that of the GRIN PC lens introduced in [185]. The experimentally obtained wave field shown in figure 33(b) validated the wave focusing capability of the developed PC formed Luneburg lens and its potential for energy harvesting efficiency improvement.

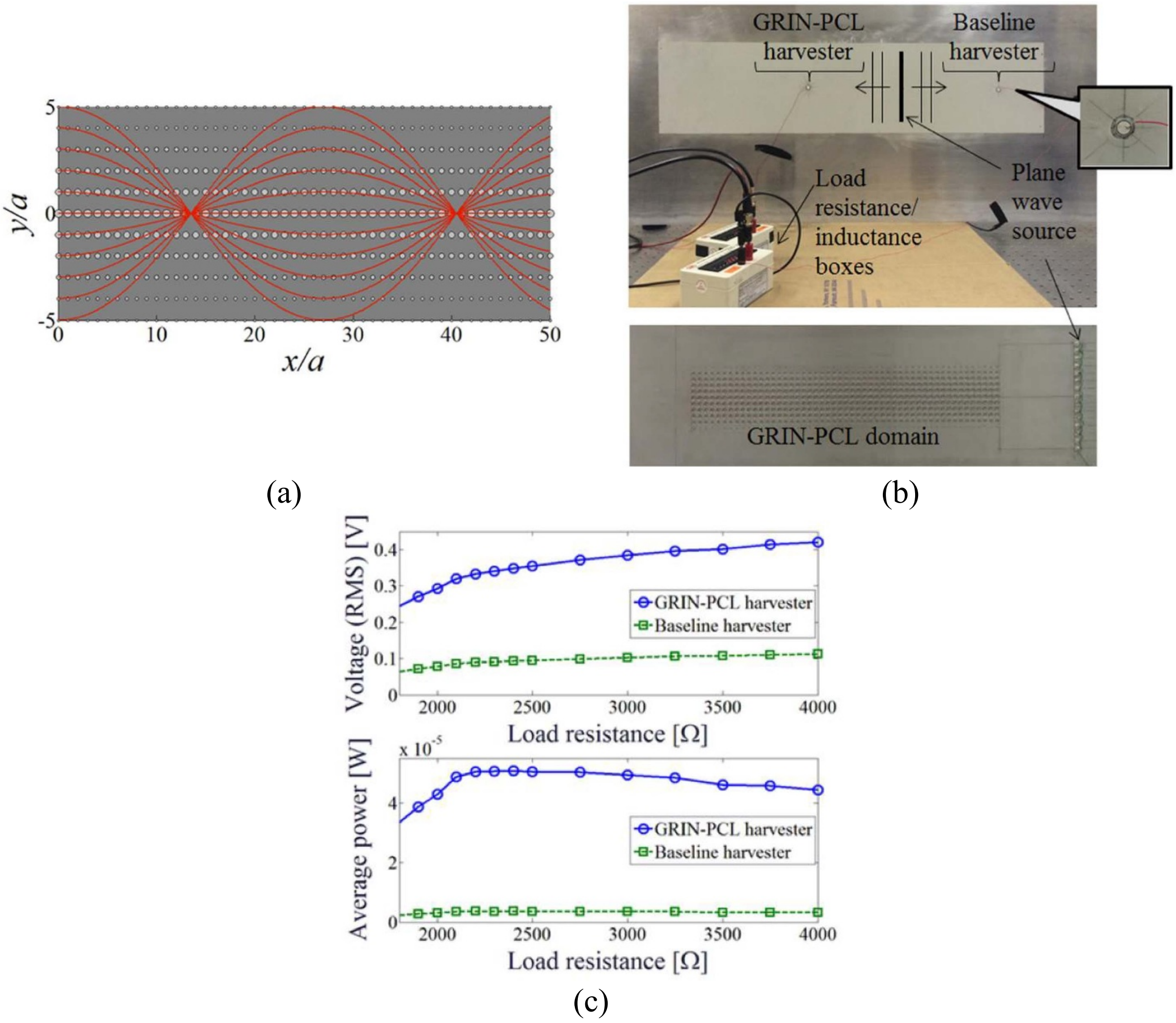

Figure 30. (a) Schematic of wave trajectory in the proposed GRIN PC lens, (b) physical prototype of the GRIN-PC lens, (c) comparison of RMS voltage and power output responses of the GRIN-PC lens based PEH and the baseline PEH without using the GRIN-PC lens at 50 kHz [185]. Reprinted from [185], with the permission of AIP Publishing.

Granular crystals, or alternatively termed as granular metamaterials, have inherent nonlinearity. A granular crystal is often constituted by a chain of particles such as sphere beads, cylinder rods etc. The stress waves in the granular crystal propagate through the contact of neighbouring particles. A recent review paper collects relevant literature on the topic of wave propagations in granular metamaterials [195]. Since the impact between the particles in the granular crystal gives rise to the generation of solitary waves which become bulk after propagating into a linear medium, Li and Rizzo [194] proposed a symmetric configuration to enable the bulk waves to coalesce into a focal spot located along the axis of symmetry for energy harvesting. It is worth mentioning that though the proposed system can also focus the waves to a focal spot, the wave focusing functionality is not realized directly using the granular crystal itself. Figure 34(a) shows the symmetric configuration of the granular crystal. The linear medium is made of a polycarbonate. Figure 34(b) illustrates the basic mechanism as already explained for energy harvesting. A piezoelectric transducer is bonded at the bottom of the linear medium, where the focal spot is formed for energy transduction. Figure 34(c) presents the output power from the piezoelectric transducer. They found that using more chains of granular crystals can increase power output. However, the harvested power level is only at the nanowatt level. Considering the large size of the whole structure, the power density of this system is actually very low. For the same system, the researchers from the same group then conducted a parametric study based on a finite element model to optimize the power output from the piezoelectric energy harvester [196, 197]. Through a proper selection of the system parameters such as the materials and size of the beads, the material of the linear medium, and the setting of the boundary condition of the piezoelectric transducer, the power output was increased to the level of milliwatt. A further parametric study of the same system through experiments has also been performed by the same group [198].

In summary, the wave focusing mechanism enhanced configurations possess mainly two advantages. First, due to the

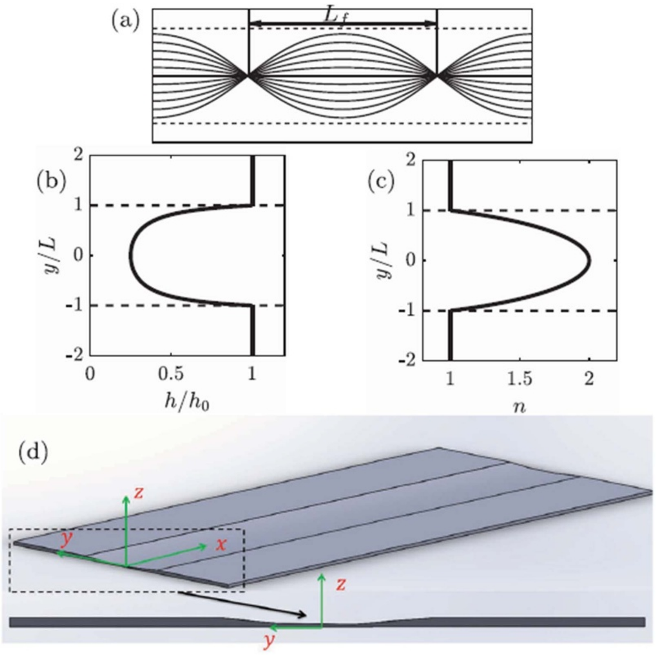

Figure 31. (a) Top view of the wave trajectory, (b) the plate thickness profile, (c) the refractive index profile, (d) the corresponding plate designed with the engineered thickness profile for obtaining the desired refractive index profile [190]. Reprinted from [190], with the permission of AIP Publishing.

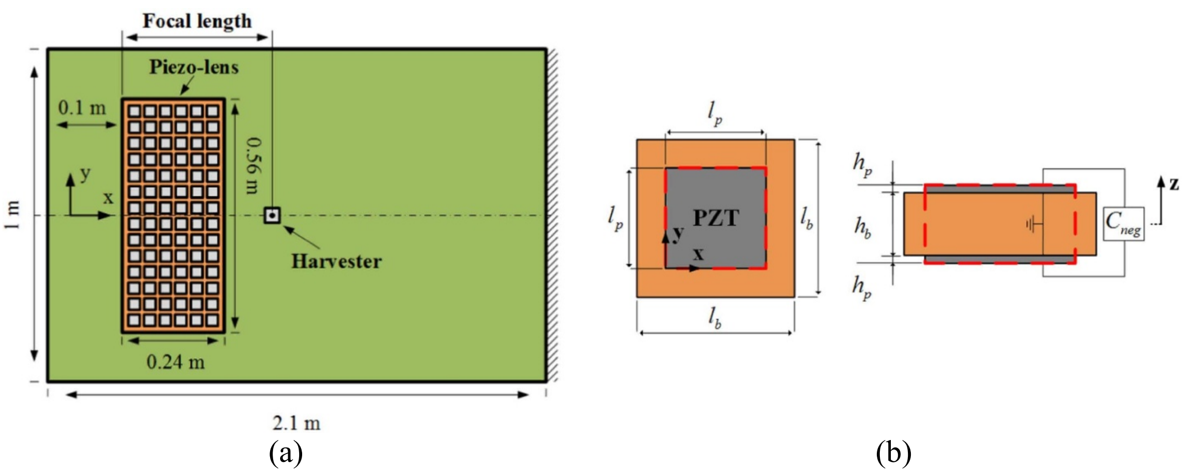

Figure 32. (a) An active PC formed lens for energy harvesting, the inclusions of the PC are realized using piezoelectric patches that are shunted to negative capacitances, (b) the top and side view of a unit cell containing a piezoelectric patch shunted to the negative capacitance [191].

wave focusing mechanism through the mirror effect or the lens effect, the waves can be well concentrated in favour of the energy harvesting purpose. Second, different from the defect state based configurations with narrow operating bandwidths, wave focusing can be realized over a broad frequency range. Hence, wave focusing mechanism enhanced configurations often have a wide operating bandwidth. Nevertheless, there are also some disadvantages of wave focusing mechanism enhanced configurations. For example, though the operating bandwidths are satisfactory, the operating frequency range is very high, making it difficult for low-frequency energy harvesting. Moreover, to achieve the mirror effect or the lens effect, the design of the refractive index profile, as well as the manufacturing of the resultant structure, is relatively complicated. In addition, taking account of the metamaterial/PC formed mirrors or lens, which play the role of energy amplifiers rather than direct energy converters, the dimension of the whole system is larger than conventional energy harvesters. This characteristic poses a challenge for the miniaturization of energy harvesting systems. Furthermore, the use of such configuration may be challenged by the question like, 'is there any plus value of the metamaterial/PC mirrors and lens besides the role as energy amplifiers?'

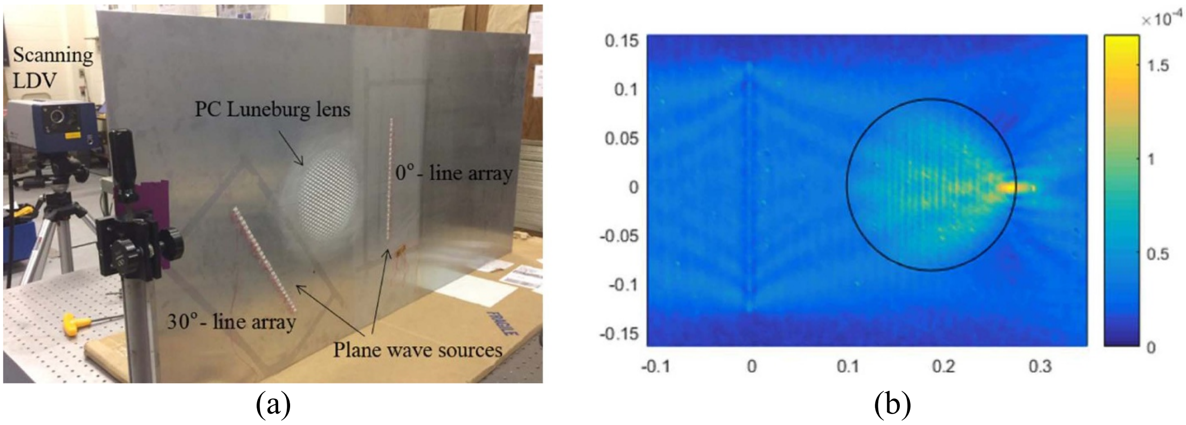

Figure 33. (a) PC formed Luneburg lens for energy harvesting, (b) experimental measured RMS wave field for demonstrating the wave focusing phenomenon [193]. Reprinted from [193], with the permission of AIP Publishing.

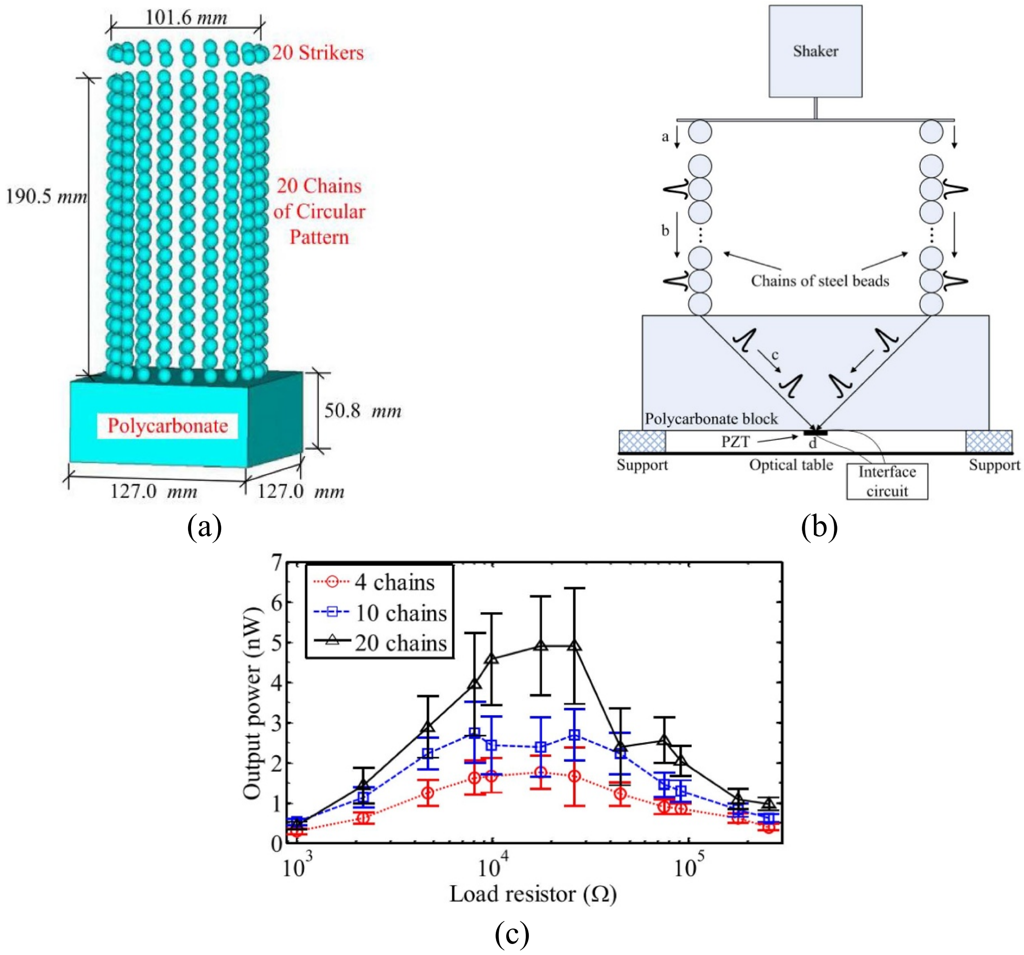

Figure 34. (a) The granular crystal consists of a chain of beads, (b) the setup for energy harvesting, (c) the output power versus the load resistance for different number of bead chains [194]. Reprinted from [194], with the permission of AIP Publishing.

6. Summaries, remarks, and outlook

Acoustic-elastic metamaterials with unique properties have been widely employed in various circumstances, including energy harvesting over the past decade. Conversion efficiency and operating bandwidth are the main concerns that limit the applications of energy harvesters in practical scenarios. Based on the characteristics of metamaterials, researchers have explored their potentials for being used in energy harvesting to achieve performance improvement. This article has reviewed and summarized the existing design approaches and classified them into three categories: directly integrated multifunctional configuration, defect state based configuration, and wave focusing mechanism enhanced configuration.

Table 1. Summary of various configurations of metamaterial-based energy harvesters in the existing literature over the past decade and comparison of their main features including such as energy level, efficiency improvement, working frequency range etc. In the 'wave type' column, 'E' and 'A' stand for elastic and acoustic, respectively. In the 'research methodology' column, 'T', 'S' and 'E' are the prefix letters of 'theoretical', 'simulation' and 'experiment', respectively.

	Reference	Power level	Efficiency improvement	Working frequency	Geometric size	Type	Transduction method	Wave type	Research method
Directly integrated multifunctional configuration	Gonella *et al* [109]	μW	NA	NA	7.5×87 mm	LR	Piezo	E	E
	Chen *et al* [118]	NA	2 times	<100 Hz	10 cells, $a = 0.1$ m	PC	PZT-5 H	E	TE
	Mikoshiba *et al* [110]	mW	NA	Around 140 Hz	7 cells, $a = 38.1$ mm	LR	ElectroMagn	E	E
	Zhang and Wu [111]	NA	NA	$40 \sim 250$ Hz	Around 10 mm	LR	PZT	E	S
	Ahmed and Banerjee [112–114]	μW	NA	$0.3 \sim 1.26$ kHz	$a = 6 \sim 20$ mm	LR	Piezo	E	SE
	Shen *et al* [94]	NA	NA	<500 Hz	3 cells, $a = 80$ mm	LR	Piezo	E	SE
	Li *et al* [139]	nW	15.3%	Around 400 Hz	1 cell	LR	PVDF	A	E
	Hu *et al* [115]	NA	NA	Dimensionless	NA	LR	Piezo	E	T
	Hu *et al* [119, 121, 122, 201]	μW	NA	<100 Hz	5 cells, $a = 30$ mm	LR	PZT	E	TS
	Li *et al* [131]	μW	NA	$146 \sim 171$ Hz	NA	LR	Piezo	E	SE
	Sugino and Erturk [120, 125]	NA	NA	Dimensionless	NA	LR	Piezo	E	T
	Sun *et al* [148]	μW	6.32	600 Hz	$a = 10$ mm	LR	Piezo	A	TSE
	Guo *et al* [146]	nW	NA	$600 \sim 950$ Hz	$a = 0.1$ m	LR	Piezo	A	TSE
	Liu *et al* [30]	nW	NA	$460 \sim 680$ Hz	$a = 20$ mm	LR	PZT-5 H	A	SE
	Mir *et al* [116, 202]	mW	NA	Around 450 Hz	$38 \times 38 \times 25$ mm^3	LR	Piezo	A	SE
	Yuan *et al* [140]	mW	NA	155 Hz	37 mm	LR	PZT-5 H	A	SE
	Zhang *et al* [145]	nW	14 times	343 and 402 Hz	1 cell, 60 mm	LR	PVDF	A	SE
	Chen *et al* [124]	μW	2 times	348 Hz	9 cells, $a = 45$ mm	LR	PVDF	E	TSE
	Fan *et al* [149]	μW	66 times	4045 Hz	$a = 80$ mm	SC	PZT-5 H	A	S
	Jin *et al* [147]	μW	NA	1303 Hz	$a = 42$ mm	LR	PZT	A	TS
	Wang *et al* [144]	nW	4.2 times	Hundreds of Hz	17 mm	LR	PZT	A	SE
	Chen *et al* [137]	NA	NA	189.3 Hz	4×4 cells	LR	PZT-5 H	E	TE
	Anigbogu and Bardaweel [136]	mW	NA	245 Hz	3×3 cells	LR	ElectroMagn	E	TSE

(Continued.)

Table 1. (Continued.)

		Reference	Power level	Efficiency improvement	Working frequency	Geometric size	Type	Transduction method	Wave type	Research method
Defect state based configuration		Wu *et al* [163]	nW	4.94 times	4.02 kHz	5×5 cells, $a = 49$ mm	SC	PVDF	A	E
		Wang *et al* [29]	nW	NA	4.21 kHz	5×5 cells, $a = 49$ mm	SC	PVDF	A	TSE
		Carrara *et al* [167, 169]	μW	NA	35 & 63 kHz	7×6 cells	PC	Piezo	E	E
		Lv *et al* [168]	nW	177 421 times	680 Hz	5×5 cells, $a = 15.5$ mm	PC	PVDF	E	SE
		Yang *et al* [164]	μW 10 J m^{-3}	262 times	5.5 kHz	5×5 cells, $a = 36$ mm	SC	PZT	A	E
		Assouar *et al* [203]	μW	NA	2303 Hz	6×6 cells, $a = 10$ mm	LR	Pizeo	E	TS
		Qi *et al* [28]	μW	NA	2257.5 Hz	6×6 cells, $a = 60$ mm	PC	PZT-5 H	A	TS
		Hu *et al* [176]	μW	NA	Around 4.5 kHz	$a = 20$ mm	PC	Piezo	E	TS
		Oudich and Li [173]	nW	NA	640 Hz	Cavity $= 3 \times 3$cm^2	LR	Piezo	A	TS
		Aly *et al* [170]	μW	NA	4310 Hz	5×5 cells, $a = 49$ mm	PC	PVDF	E	S
		Deng *et al* [178, 179]	NA	76.67 times	Hundreds of Hz	5×5 cells	PC	PZT-5 H	A	TS
		Geng *et al* [177]	mW	NA	Hundreds of Hz	13 cells, $a = 80$ mm	PC	PVDF	E	T
		Park *et al* [175]	mW	20 times	50 kHz	$a = 34$ mm	PC	PZT	E	SE
		Jo *et al* [172, 174]	μW	NA	$50 \sim 60$ kHz	5×6 cells, $a = 33$ mm	PC	PZT-4D	E	S
		Ma *et al* [204]		950%	$3000 \sim 4000$ Hz	6×6 cells, $a = 10$ mm	PC	PZT-5 H	A	SE
Wave focusing mechanism enhanced configuration	Mirror type	Carrara *et al* [180]	μW	3075%	$25 \sim 150$ kHz	100×100 mm	PC	Piezo Disk	E	SE
		Tol *et al* [181]	NA	NA	$30 \sim 70$ kHz	NA	PC	Piezo	E	S
		Darabi and Leamy [183]	mW	2-orders of magn	kHz level	cm level	PC	Piezo	E	TE
		Qi and Assouar [184]	μW	407 times	3430 Hz	$a = 12.5$ mm	SC	Piezo	A	TS
		Tol *et al* [182]	μW	11 times	$30 \sim 70$ kHz	120 mm	PC	Piezo	E	SE
	Lens type	Li and Rizzo [194, 196–198]	mW	NA	NA	190.5 mm	GC	PZT	E	TSE
		Tol *et al* [185–187, 193]	mW	13.8 times	$30 \sim 70$ kHz	$a = 8$ mm	PC	Piezo	E	SE
		Tol *et al* [192]	NA	NA	Around 400 Hz	$a = 20$ mm	LR	Piezo	E	S
		Yi *et al* [191]	NA	3 times	Around 2000 Hz	0.24×0.56 cm^2	PC	Piezo	E	S
		Zareei *et al* [190]	μW	2-orders of magn	$10 \sim 80$ kHz	50 cm	NA	Piezo	E	E
		Hyun *et al* [189]	mW	3.8 times	50 kHz	$a = 5$mm	PC	Piezo	E	SE

Table 1 lists most of the literature reviewed in this paper and sorts them into the defined three categories. Table 1 also provides a comparison of the main features, such as the power level, efficiency improvement, working frequency range, of the reviewed systems. For the wave type column, 'E' and 'A' stand for elastic and acoustic, respectively, indicating that this work is focused on harvesting energy from elastic or acoustic waves. The last column 'research methodology' is used to indicate what methods have been adopted in this research. 'T', 'S' and 'E' are the prefix letters of 'theoretical', 'simulation' and 'experiment', respectively. For instance, a research being classified into the 'TS' type implies that both theoretical and simulation studies have been conducted in this research.

From table 1, we can find that the directly integrated multifunctional configurations have been most extensively exploited. For these configurations, the main focus of researchers is on how to design a proper mechanical structure to enable the installation of transducers for realizing energy conversion. Interestingly, almost all the existing designs adopted the piezoelectric transducers. This may be due to the ease of implementation and integration of piezoelectric transducers into systems. Using other transduction mechanisms such as the electromagnetic one, one may need to devote lots of efforts to revising the mechanical architecture to contain magnets and coils. Another potential reason is that piezoelectric transducers are suitable for being used in the circumstances with small vibration displacements but large stresses. The power outputs of electromagnetic transducers are strongly dependent on the oscillating velocity. Therefore, the piezoelectric type transduction is advantageous over the electromagnetic type to be installed in metamaterials where the vibrations are significantly suppressed. From this perspective, the electret-based electrostatic transduction [43], which does not require large displacement excitations, might also be applicable in this circumstance to corporate with metamaterials concurrently. Nevertheless, no related research has been found until now. The potential reason is that the electret-based electrostatic transductions are mostly used in MEMS scale energy harvester, while the current research is still limited to the regular size of metamaterials at the scale larger than centimetres. With the development of elastic metamaterials in the future, researchers may explore the feasibility of combining MEMS scale metamaterial with electret-based electrostatic transducers for energy harvesting or self-powered sensing, etc. Besides the transduction mechanism problem, as we aforementioned that though the development of both metamaterials and vibration energy harvesting have entered the era of nonlinearity, the research of metamaterial-based energy harvesters is still confined in the linear scope. What the recipe of nonlinear metamaterials and nonlinear energy harvesters will bring to us is still unknown. Moreover, as one of the popular branches of metamaterials, topological metamaterials with some interesting phenomena, such as interface modes, have not been fully explored for energy harvesting. Therefore, future research in this direction may pay more attention to the development of topological metamaterials and try to unearth their potentials for energy harvesting.

As revealed in table 1, from the perspective of efficiency improvement, the latter two types of configurations seem to be more effective. This is actually easily understandable and expectable, since both the defect state mode and the wave focusing mechanism have the capability to localize the energy. However, in terms of the defect state based configurations, there are two major drawbacks. One is that the working frequency is very high, which can not cater for the demand of harvesting energy from low-frequency environmental vibrations. The other drawback is that the operating bandwidth is narrow. Similar to a traditional linear vibration energy harvester, a defect state-based configuration can only deliver substantial power output around the eigen-frequency of the defect mode. The first drawback may be partly overcome by replacing currently widely used PCs with locally resonant metamaterials [173], since metamaterials have been proved to have the capability to operate in relatively lower frequency ranges. In terms of the second drawback, an intuitive thought is that tunable metamaterials/PCs could be a potential solution. However, most active tunable systems require external power supplies, which are contradictory to our original intention for harvesting energy. For this reason, we think that passively self-adaptive techniques could be a potential solution. Assuming that the defected local resonator has the self-adaptive ability, the defect mode may also exhibit a self-adaption behaviour and can cover a certain range of frequencies, which can then be used for energy harvesting. As compared to the defect state based configuration, the wave focusing mechanism enhanced configuration also has the advantage of high-efficiency improvement but does not have the narrow bandwidth drawback. However, the high working frequency is still the drawback of this configuration. Since most of the existing designs of such kind of configurations were based on PCs and considered the lattice constants normally at the scale of several centimetres, we can understand the reason why their working frequencies were often at the kHz level. Using locally resonant metamaterials may improve this situation by significantly reducing the working frequency, which has been preliminary proved by the work of Tol *et al* [192]. However, further efforts need to be devoted to promoting this work.

On the other hand, most of existing research focused on the use of metamaterials for energy harvesting, mainly from the mechanical perspective. As well-known that the design of any energy harvesting systems involves the knowledge of multiple disciplines. The interface circuit could also play an important role in affecting the energy harvesting performance of a system. Therefore, according to the characteristics of metamaterial-based energy harvesters, does there exist the possibility to develop more advanced or suitable interface circuits, specially, for these types of energy harvesters to boost their energy conversion efficiencies? Though a recent study by [137] made an early attempt to introduce advanced interface circuits in metamaterial-based energy harvesting, the research in this potential direction is quite deficient. In addition, because of the periodic arrangement of electromechanical transducers inside metamaterials, the power manangement of the multiple different outputs becomes a challenge and a difficulty [199, 200]. Therefore, besides of advanced interface circuits for efficiency boosting, there may need a special power manangement unit for energy optimization.

In general, metamaterial-based energy harvesting has a bright future for applications in such as smart building blocks for vibration isolation and noise reduction, powering structural health monitoring sensors embedded in the infrastructure, and vibration/shock absorbers with energy recycling abilities for cars and trains etc. However, the currently developed systems are still limited at the conceptual design stage. The authors hope that this review article can provide a comprehensive and helpful summary of the existing work to the researchers who have interests in this direction. Moreover, from our defined classifications of existing design approaches, the researchers can find some guidance for developing a metamaterial-based energy harvester. In addition, we have made some comments and remarks on the disadvantages of the existing designs and put forward prospects on what can be done in the future to find potential solutions. We hope that the researchers can be inspired to make their contributions to the development of this research direction. It is envisioned that someday metamaterial-based energy harvesters will be deployed in practical scenarios to suppress undesired vibrations and sound noise, and concurrently convert the inherent vibration energy into electrical energy for providing a sustainable power solution.

ORCID iDs

Guobiao Hu https://orcid.org/0000-0002-1288-7564
Lihua Tang https://orcid.org/0000-0001-9031-4190
Junrui Liang https://orcid.org/0000-0003-2685-5587
Chunbo Lan https://orcid.org/0000-0001-5959-0488
Raj Das https://orcid.org/0000-0001-9977-6201

References

[1] Gilbert J M and Balouchi F 2008 Comparison of energy harvesting systems for wireless sensor networks *Int. J. Autom. Comput.* **5** 334–47
[2] Mateu L and Moll F 2005 Review of energy harvesting techniques and applications for microelectronics (Keynote Address) *Proc. SPIE*
[3] Priya S and Inman D J 2009 *Energy Harvesting Technologies* vol 21 (Berlin: Springer)
[4] Wang J, Geng L, Ding L, Zhu H and Yurchenko D 2020 The state-of-the-art review on energy harvesting from flow-induced vibrations *Appl. Energy* **267** 114902
[5] Liu H, Zhong J, Lee C, Lee S-W and Lin L 2018 A comprehensive review on piezoelectric energy harvesting technology: materials, mechanisms, and applications *Appl. Phys. Rev.* **5** 041306
[6] Kim M, Dugundji J and Wardle B L 2015 Efficiency of piezoelectric mechanical vibration energy harvesting *Smart Mater. Struct.* **24** 055006
[7] Gong L J, Pan Q S, Li W, Yan G Y, Liu Y B and Feng Z H 2015 Harvesting vibration energy using two modal vibrations of a folded piezoelectric device *Appl. Phys. Lett.* **107** 033904
[8] Boisseau S, Despesse G and Seddik B A 2013 Nonlinear H-shaped springs to improve efficiency of vibration energy harvesters *J. Appl. Mech.-Trans. ASME* **80** 061013
[9] Harne R L and Wang K W 2013 A review of the recent research on vibration energy harvesting via bistable systems *Smart Mater. Struct.* **22** 023001
[10] Soliman M S M, Abdel-Rahman E M, El-Saadany E F and Mansour R R 2008 A wideband vibration-based energy harvester *J. Micromech. Microeng.* **18** 115021
[11] Elvin N and Erturk A 2013 *Advances in Energy Harvesting Methods* (New York: Springer Science & Business Media)
[12] Berdy D F, Srisungsitthisunti P, Jung B, Xu X, Rhoads J F and Peroulis D 2012 Low-frequency meandering piezoelectric vibration energy harvester *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **59** 846–58
[13] Choi W J, Jeon Y, Jeong J H, Sood R and Kim S G 2006 Energy harvesting MEMS device based on thin film piezoelectric cantilevers *J. Electroceramics* **17** 543–8
[14] Ibrahim M and Salehian A 2014 Modeling, fabrication, and experimental validation of hybrid piezo-magnetostrictive and piezomagnetic energy harvesting units *J. Intell. Mater. Syst. Struct.* **52** 584–95
[15] Karami M A and Inman D J 2011 Analytical modeling and experimental verification of the vibrations of the zigzag microstructure for energy harvesting *J. Vib. Acoust.-Trans. ASME* **133** 011002
[16] Lueke J, Rezaei M and Moussa W 2014 Investigation of folded spring structures for vibration-based piezoelectric energy harvesting *J. Micromech. Microeng.* **24** 125011
[17] Aly A H and Mohamed D 2019 The optical properties of metamaterial-superconductor photonic band gap with/without defect layer *J. Supercond. Novel Magnet.* **32** 1897–902
[18] Aly A H, Elsayed H A and El-Naggar S A 2017 Tuning the flow of light in two-dimensional metallic photonic crystals based on Faraday effect *J. Mod. Opt.* **64** 74–80
[19] Hussein M I, Leamy M J and Ruzzene M 2014 Dynamics of phononic materials and structures: historical origins, recent progress, and future outlook *Appl. Mech. Rev.* **66** 040802
[20] Kushwaha M S, Halevi P, Martinez G, Dobrzynski L and Djafari-Rouhani B 1994 Theory of acoustic band structure of periodic elastic composites *Phys. Rev.* B **49** 2313
[21] Lu M, Feng L and Chen Y 2009 Phononic crystals and acoustic metamaterials *Mater. Today* **12** 34–42
[22] Shalaev V M, Cai W, Chettiar U K, Yuan H K, Sarychev A K, Drachev V P and Kildishev A V 2005 Negative index of refraction in optical metamaterials *Opt. Lett.* **30** 3356–8
[23] Yao J, Liu Z, Liu Y, Wang Y, Sun C, Bartal G, Stacy A M and Zhang X 2008 Optical negative refraction in bulk metamaterials of nanowires *Science* **321** 930
[24] Huang H and Sun C 2012 Anomalous wave propagation in a one-dimensional acoustic metamaterial having simultaneously negative mass density and Young's modulus *J. Acoust. Soc. Am.* **132** 2887–95
[25] Liu X, Hu G, Huang G and Sun C 2011 An elastic metamaterial with simultaneously negative mass density and bulk modulus *Appl. Phys. Lett.* **98** 251907
[26] Yao S, Zhou X and Hu G 2008 Experimental study on negative effective mass in a 1D mass-spring system *New J. Phys.* **10** 043020
[27] Huang H, Sun C and Huang G 2009 On the negative effective mass density in acoustic metamaterials *Int. J. Eng. Sci.* **47** 610–7
[28] Qi S, Oudich M, Li Y and Assouar B 2016 Acoustic energy harvesting based on a planar acoustic metamaterial *Appl. Phys. Lett.* **108** 263501
[29] Wang W-C, Wu L-Y, Chen L-W and Liu C-M 2010 Acoustic energy harvesting by piezoelectric curved beams in the cavity of a sonic crystal *Smart Mater. Struct.* **19** 045016
[30] Liu G-S, Peng -Y-Y, Liu M-H, Zou X-Y and Cheng J-C 2018 Broadband acoustic energy harvesting metasurface with

coupled Helmholtz resonators *Appl. Phys. Lett.* **113** 153503
[31] Chen Z, Guo B, Yang Y and Cheng C 2014 Metamaterials-based enhanced energy harvesting: a review *Physica* B **438** 1–8
[32] Ahmed R, Mir F and Banerjee S 2017 A review on energy harvesting approaches for renewable energies from ambient vibrations and acoustic waves using piezoelectricity *Smart Mater. Struct.* **26** 085031
[33] Tan T, Yan Z, Zou H, Ma K, Liu F, Zhao L, Peng Z and Zhang W 2019 Renewable energy harvesting and absorbing via multi-scale metamaterial systems for internet of things *Appl. Energy* **254** 113717
[34] Liang J and Liao W 2009 Piezoelectric energy harvesting and dissipation on structural damping *J. Intell. Mater. Syst. Struct.* **20** 515–27
[35] Lesieutre G A, Ottman G K and Hofmann H F 2004 Damping as a result of piezoelectric energy harvesting *J. Sound Vib.* **269** 991–1001
[36] Moheimani S R and Fleming A J 2006 *Piezoelectric Transducers for Vibration Control and Damping* (New York: Springer Science & Business Media)
[37] Hagood N W and Von Flotow A 1991 Damping of structural vibrations with piezoelectric materials and passive electrical networks *J. Sound Vib.* **146** 243–68
[38] Chen S, Wen J, Yu D, Wang G and Wen X 2011 Band gap control of phononic beam with negative capacitance piezoelectric shunt *Chin. Phys.* B **20** 014301
[39] Chen Y, Huang G and Sun C 2014 Band gap control in an active elastic metamaterial with negative capacitance piezoelectric shunting *J. Vib. Acoust.* **136** 061008
[40] Zhu R, Chen Y Y, Barnhart M V, Hu G K, Sun C T and Huang G L 2016 Experimental study of an adaptive elastic metamaterial controlled by electric circuits *Appl. Phys. Lett.* **108** 011905
[41] Hu G, Xu J, Tang L, Lan C and Das R 2019 Tunable metamaterial beam using negative capacitor for local resonators coupling *J. Intell. Mater. Syst. Struct.* **31** 389–407
[42] Wang Y-F, Wang Y-Z, Wu B, Chen W and Wang Y-S 2020 Tunable and active phononic crystals and metamaterials *Appl. Mech. Rev.* **72** 040801
[43] Tao K, Tang L, Wu J, Lye S W, Chang H and Miao J 2018 Investigation of multimodal electret-based MEMS energy harvester with impact-induced nonlinearity *J. Microelectromech. Syst.* **27** 276–88
[44] Zhou S and Wang J 2018 Dual serial vortex-induced energy harvesting system for enhanced energy harvesting *AIP Adv.* **8** 075221
[45] Fang S, Fu X, Du X and Liao W-H 2019 A music-box-like extended rotational plucking energy harvester with multiple piezoelectric cantilevers *Appl. Phys. Lett.* **114** 233902
[46] Hu G, Tang L and Das R 2017 An impact-engaged two-degrees-of-freedom piezoelectric energy harvester for wideband operation *Proc. Eng.* **173** 1463–70
[47] Arroyo E, Badel A, Formosa F, Wu Y and Qiu J 2012 Comparison of electromagnetic and piezoelectric vibration energy harvesters: model and experiments *Sens. Actuators* A **183** 148–56
[48] Zhang Y, Wang T, Luo A, Hu Y, Li X and Wang F 2018 Micro electrostatic energy harvester with both broad bandwidth and high normalized power density *Appl. Energy* **212** 362–71
[49] 1996 IEEE standard on piezoelectricity *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **43** A1–A54
[50] Priya S 2007 Advances in energy harvesting using low profile piezoelectric transducers *J. Electroceramics* **19** 167–84
[51] Beeby S P, Tudor M J and White N 2006 Energy harvesting vibration sources for microsystems applications *Meas. Sci. Technol.* **17** R175
[52] Saha C R, O'Donnell T, Loder H, Beeby S and Tudor J 2006 Optimization of an electromagnetic energy harvesting device *IEEE Trans. Magn.* **42** 3509–11
[53] Torres E O and Rincón-Mora G A 2008 Electrostatic energy-harvesting and battery-charging CMOS system prototype *IEEE Trans. Circ. Syst.* I **56** 1938–48
[54] Tan Y K and Panda S K 2010 Review of energy harvesting technologies for sustainable wireless sensor network *Sustainable Wireless Sensor Networks* (London: IntechOpen) pp 1–30
[55] Roundy S, Wright P K and Rabaey J 2003 A study of low level vibrations as a power source for wireless sensor nodes *Comput. Commun.* **26** 1131–44
[56] Tang L H and Yang Y W 2012 A multiple-degree-of-freedom piezoelectric energy harvesting model *J. Intell. Mater. Syst. Struct.* **23** 1631–47
[57] Wang X and Lin L W 2013 Dimensionless optimization of piezoelectric vibration energy harvesters with different interface circuits *Smart Mater. Struct.* **22** 085011
[58] Xiao H, Wang X and John S 2015 A dimensionless analysis of a 2DOF piezoelectric vibration energy harvester *Mech. Syst. Signal Process.* **58–59** 355–75
[59] Guyomar D, Sebald G, Pruvost S, Lallart M, Khodayari A and Richard C 2009 Energy harvesting from ambient vibrations and heat *J. Intell. Mater. Syst. Struct.* **20** 609–24
[60] Guyomar D, Badel A, Lefeuvre E and Richard C 2005 Toward energy harvesting using active materials and conversion improvement by nonlinear processing *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **52** 584–95
[61] Lefeuvre E, Badel A, Richard C and Guyomar D 2005 Piezoelectric energy harvesting device optimization by synchronous electric charge extraction *J. Intell. Mater. Syst. Struct.* **16** 865–76
[62] Wu H, Tang L H, Yang Y W and Soh C K 2014 Development of a broadband nonlinear two-degree-of-freedom piezoelectric energy harvester *J. Intell. Mater. Syst. Struct.* **25** 1875–89
[63] Kim I-H, Jung H-J, Lee B M and Jang S-J 2011 Broadband energy-harvesting using a two degree-of-freedom vibrating body *Appl. Phys. Lett.* **98** 214102
[64] Wu H, Tang L, Yang Y and Soh C K 2013 A novel two-degrees-of-freedom piezoelectric energy harvester *J. Intell. Mater. Syst. Struct.* **24** 357–68
[65] Gammaitoni L, Neri I and Vocca H 2009 Nonlinear oscillators for vibration energy harvesting *Appl. Phys. Lett.* **94** 164102
[66] Mann B P and Owens B A 2010 Investigations of a nonlinear energy harvester with a bistable potential well *J. Sound Vib.* **329** 1215–26
[67] Xu J and Tang J 2015 Multi-directional energy harvesting by piezoelectric cantilever-pendulum with internal resonance *Appl. Phys. Lett.* **107** 213902
[68] Xiong L, Tang L and Mace B R 2016 Internal resonance with commensurability induced by an auxiliary oscillator for broadband energy harvesting *Appl. Phys. Lett.* **108** 203901
[69] Cao D, Leadenham S and Erturk A 2015 Internal resonance for nonlinear vibration energy harvesting *Eur. Phys. J. Spec. Top.* **224** 2867–80
[70] Gu L and Livermore C 2010 Passive self-tuning energy harvester for extracting energy from rotational motion *Appl. Phys. Lett.* **97** 081904
[71] Jo S, Kim M and Kim Y 2011 Passive-self-tunable vibrational energy harvester *Solid-State Sensors, Actuators and Microsystems Conf. (TRANSDUCERS), 2011 16th Int.* (IEEE)

[72] Miller L M, Pillatsch P, Halvorsen E, Wright P K, Yeatman E M and Holmes A S 2013 Experimental passive self-tuning behavior of a beam resonator with sliding proof mass *J. Sound Vib.* **332** 7142–52

[73] Liu W, Zhao C, Badel A, Formosa F, Zhu Q and Hu G 2018 Compact self-powered synchronous energy extraction circuit design with enhanced performance *Smart Mater. Struct.* **27** 047001

[74] Zhao L, Tang L and Yang Y 2016 Synchronized charge extraction in galloping piezoelectric energy harvesting *J. Intell. Mater. Syst. Struct.* **27** 453–68

[75] Shu Y C, Lien I C and Wu W J 2007 An improved analysis of the SSHI interface in piezoelectric energy harvesting *Smart Mater. Struct.* **16** 2253–64

[76] Liang J and Liao W 2012 Impedance modeling and analysis for piezoelectric energy harvesting systems *IEEE/ASME Trans. Mechatronics* **17** 1145–57

[77] Liang J and Liao W-H 2011 Improved design and analysis of self-powered synchronized switch interface circuit for piezoelectric energy harvesting systems *IEEE Trans. Ind. Electron.* **59** 1950–60

[78] Eltamaly A M and Addoweesh K E 2016 A novel self-power SSHI circuit for piezoelectric energy harvester *IEEE Trans. Power Electron.* **32** 7663–73

[79] Lu S H and Boussaid F 2015 A highly efficient P-SSHI rectifier for piezoelectric energy harvesting *IEEE Trans. Power Electron.* **30** 5364–9

[80] Liao Y and Sodano H 2018 Optimal power, power limit and damping of vibration based piezoelectric power harvesters *Smart Mater. Struct.* **27** 075057

[81] Liao Y and Liang J 2018 Maximum power, optimal load, and impedance analysis of piezoelectric vibration energy harvesters *Smart Mater. Struct.* **27** 075053

[82] Lan C, Liao Y, Hu G and Tang L 2020 Equivalent impedance and power analysis of monostable piezoelectric energy harvesters *J. Intell. Mater. Syst. Struct.* **31** 1697–715

[83] Szarka G D, Stark B H and Burrow S G 2011 Review of power conditioning for kinetic energy harvesting systems *IEEE Trans. Power Electron.* **27** 803–15

[84] Sigalas M M and Economou E N 1992 Elastic and acoustic wave band structure *J. Sound Vib.* **158** 377–82

[85] Richards D and Pines D J 2003 Passive reduction of gear mesh vibration using a periodic drive shaft *J. Sound Vib.* **264** 317–42

[86] Olsson R H III and El-Kady I 2008 Microfabricated phononic crystal devices and applications *Meas. Sci. Technol.* **20** 012002

[87] Sánchez-Dehesa J, Garcia-Chocano V M, Torrent D, Cervera F, Cabrera S and Simon F 2011 Noise control by sonic crystal barriers made of recycled materials *J. Acoust. Soc. Am.* **129** 1173–83

[88] Espinosa V, Sánchez-Morcillo V J, Staliunas K, Pérez-Arjona I and Redondo J 2007 Subdiffractive propagation of ultrasound in sonic crystals *Phys. Rev.* B **76** 140302

[89] Liu Z Y, Zhang X X, Mao Y W, Zhu Y Y, Yang Z Y, Chan C T and Sheng P 2000 Locally resonant sonic materials *Science* **289** 1734–6

[90] Zhu R, Liu X, Hu G, Sun C and Huang G 2014 A chiral elastic metamaterial beam for broadband vibration suppression *J. Sound Vib.* **333** 2759–73

[91] Zhang S W, Wu J H and Hu Z P 2013 Low-frequency locally resonant band-gaps in phononic crystal plates with periodic spiral resonators *J. Appl. Phys.* **113** 163511

[92] Daniel T and José S-D 2008 Acoustic cloaking in two dimensions: a feasible approach *New J. Phys.* **10** 063015

[93] Bigoni D, Guenneau S, Movchan A and Brun M 2013 Elastic metamaterials with inertial locally resonant structures: application to lensing and localization *Phys. Rev.* B **87** 174303

[94] Shen L, Wu J, Zhang S, Liu Z and Li J 2015 Low-frequency vibration energy harvesting using a locally resonant phononic crystal plate with spiral beams *Modern Phys. Lett.* B **29** 1450259

[95] Kushwaha M and Halevi P 1996 Giant acoustic stop bands in two-dimensional periodic arrays of liquid cylinders *Appl. Phys. Lett.* **69** 31–3

[96] Laude V, Wilm M, Benchabane S and Khelif A 2005 Full band gap for surface acoustic waves in a piezoelectric phononic crystal *Phys. Rev.* E **71** 036607

[97] Wang Y, Li F, Wang Y, Kishimoto K and Huang W 2009 Tuning of band gaps for a two-dimensional piezoelectric phononic crystal with a rectangular lattice *Acta Mech. Sin.* **25** 65–71

[98] Djafari-Rouhani B. Phononic crystals: fundamental and applications *26th Janik's Friends Meeting, Neutron Scattering Spectroscopy and Related Problems*

[99] Kushwaha M S and Djafari-Rouhani B 1998 Giant sonic stop bands in two-dimensional periodic system of fluids *J. Appl. Phys.* **84** 4677–83

[100] Pennec Y, Vasseur J O, Djafari-Rouhani B, Dobrzyński L and Deymier P A 2010 Two-dimensional phononic crystals: examples and applications *Surf. Sci. Rep.* **65** 229–91

[101] Benchabane S, Khelif A, Rauch J-Y, Robert L and Laude V 2006 Evidence for complete surface wave band gap in a piezoelectric phononic crystal *Phys. Rev.* E **73** 065601

[102] Hou Z, Wu F and Liu Y 2004 Phononic crystals containing piezoelectric material *Solid State Commun.* **130** 745–9

[103] Nouh M, Aldraihem O and Baz A 2014 Metamaterial structures with periodic local resonances *Proc. SPIE* **9064** 90641Y

[104] Nouh M, Aldraihem O and Baz A 2015 Wave propagation in metamaterial plates with periodic local resonances *J. Sound Vib.* **341** 53–73

[105] Milton G W and Willis J R 2007 On modifications of Newton's second law and linear continuum elastodynamics *Proc. R. Soc.* A **463** 855–80

[106] Banerjee B 2011 *An Introduction to Metamaterials and Waves in Composites* (Boca Raton, FL: CRC Press)

[107] Banerjee A, Das R and Calius E P 2017 Frequency graded 1D metamaterials: a study on the attenuation bands *J. Appl. Phys.* **122** 075101

[108] Fang N, Xi D, Xu J, Ambati M, Srituravanich W, Sun C and Zhang X 2006 Ultrasonic metamaterials with negative modulus *Nat. Mater.* **5** 452–6

[109] Gonella S, To A C and Liu W K 2009 Interplay between phononic bandgaps and piezoelectric microstructures for energy harvesting *J. Mech. Phys. Solids* **57** 621–33

[110] Mikoshiba K, Manimala J M and Sun C 2013 Energy harvesting using an array of multifunctional resonators *J. Intell. Mater. Syst. Struct.* **24** 168–79

[111] Zhang S and Wu J H 2013 Low-frequency broadband energy harvesting based on locally resonant phononic crystals *ASME 2013 Int. Mechanical Engineering Congress and Exposition* (American Society of Mechanical Engineers Digital Collection)

[112] Ahmed R U and Banerjee S 2014 Low frequency energy scavenging using sub-wave length scale acousto-elastic metamaterial *AIP Adv.* **4** 117114

[113] Ahmed R U, Adiba A and Banerjee S 2015 Energy scavenging from acousto-elastic metamaterial using local resonance phenomenon *Proc. SPIE*

[114] Ahmed R, Madisetti D and Banerjee S 2017 A sub-wavelength scale acoustoelastic sonic crystal for harvesting energies at very low frequencies (<∼1 kHz) using controlled geometric configurations *J. Intell. Mater. Syst. Struct.* **28** 381–91

[115] Hu G, Tang L, Banerjee A and Das R 2017 Metastructure with piezoelectric element for simultaneous vibration suppression and energy harvesting *J. Vib. Acoust.* **139** 011012

[116] Mir F, Saadatzi M S, Ahmed R U and Banerjee S 2018 The possibility of harvesting electrical energy from industrial noise barriers using meta-wall bricks *Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems 2018* (International Society for Optics and Photonics)

[117] Saadatzi M, Mir F, Saadatzi M N, Tavaf V and Banerjee S 2018 Modeling of a 3D acoustoelastic metamaterial energy harvester *Active and Passive Smart Structures and Integrated Systems XII* (International Society for Optics and Photonics)

[118] Chen Z, Yang Y, Lu Z and Luo Y 2013 Broadband characteristics of vibration energy harvesting using one-dimensional phononic piezoelectric cantilever beams *Physica* B **410** 5–12

[119] Hu G, Tang L and Das R 2017 Metamaterial-inspired piezoelectric system with dual functionalities: energy harvesting and vibration suppression *Active and Passive Smart Structures and Integrated Systems 2017* (International Society for Optics and Photonics)

[120] Sugino C, Guillot V and Erturk A 2017 Multifunctional energy harvesting locally resonant metastructures *ASME 2017 Conf. on Smart Materials, Adaptive Structures and Intelligent Systems* (American Society of Mechanical Engineers Digital Collection)

[121] Hu G, Tang L and Das R 2018 Internally coupled metamaterial beam for simultaneous vibration suppression and low frequency energy harvesting *J. Appl. Phys.* **123** 055107

[122] Hu G, Tang L and Das R 2018 General framework for modeling multifunctional metamaterial beam based on a derived one-dimensional piezoelectric composite finite element *J. Aerosp. Eng.* **31** 04018088

[123] Hu Y, Hu H and Yang J 2006 A low frequency piezoelectric power harvester using a spiral-shaped bimorph *Sci. China Ser.* G **49** 649–59

[124] Chen J-S, Su W-J, Cheng Y, Li W-C and Lin C-Y 2019 A metamaterial structure capable of wave attenuation and concurrent energy harvesting *J. Intell. Mater. Syst. Struct.* **30** 2973–81

[125] Sugino C and Erturk A 2018 Analysis of multifunctional piezoelectric metastructures for low-frequency bandgap formation and energy harvesting *J. Phys. D: Appl. Phys.* **51** 215103

[126] Huang G and Sun C 2010 Band gaps in a multiresonator acoustic metamaterial *J. Vib. Acoust.-Trans. ASME* **132** 031003

[127] Emerson T, Lozzi A, Bai H and Manimala J 2018 Dynamic characterization and control of a metamaterials-inspired smart composite *ASME 2018 Conf. on Smart Materials, Adaptive Structures and Intelligent Systems* (American Society of Mechanical Engineers Digital Collection)

[128] Dwivedi A, Banerjee A and Bhattacharya B 2019 Study of piezo embedded negative mass metamaterial using generalized Bloch theorem for energy harvesting system *Active and Passive Smart Structures and Integrated Systems XII* (International Society for Optics and Photonics)

[129] Mir F, Indaleeb M M, Ahmed H and Banerjee S 2019 Study of split ring metamaterial with simultaneous wave guiding and energy harvesting capability *Active and Passive Smart Structures and Integrated Systems XII* (International Society for Optics and Photonics)

[130] Dwivedi A, Banerjee A and Bhattacharya B 2020 Simultaneous energy harvesting and vibration attenuation in piezo-embedded negative stiffness metamaterial *J. Intell. Mater. Syst. Struct.* **31** 1076–90

[131] Li Y, Baker E, Reissman T, Sun C and Liu W K 2017 Design of mechanical metamaterials for simultaneous vibration isolation and energy harvesting *Appl. Phys. Lett.* **111** 251903

[132] Ziniu L, Zhongsheng C, Fan Z and Xiong Y 2017 Effects of geometric and material parameters on band-gaps of piezoelectric vibration energy harvesting plate with local resonators *2017 Prognostics and System Health Management Conf. (Phm-harbin)* (IEEE)

[133] Chen Z, He J and Wang G 2019 Vibration bandgaps of piezoelectric metamaterial plate with local resonators for vibration energy harvesting *Shock Vib.* **2019** 1–15

[134] Lv X-F, Fang X, Zhang Z-Q, Huang Z-L and Chuang K-C 2019 Highly localized and efficient energy harvesting in a phononic crystal beam: defect placement and experimental validation *Crystals* **9** 391

[135] Hobeck J D and Inman D J Sr 2017 Simultaneous passive broadband vibration suppression and energy harvesting with multifunctional metastructures *A Tribute Conf. Honoring Daniel Inman* (International Society for Optics and Photonics)

[136] Anigbogu W and Bardaweel H 2020 A metamaterial-inspired structure for simultaneous vibration attenuation and energy harvesting *Shock Vib.* **2020** 1–12

[137] Chen Z, Xia Y, He J, Xiong Y and Wang G 2020 Elastic-electro-mechanical modeling and analysis of piezoelectric metamaterial plate with a self-powered synchronized charge extraction circuit for vibration energy harvesting *Mech. Syst. Signal Process.* **143** 106824

[138] Chen C, Zhao B and Liang J 2019 Revisit of synchronized electric charge extraction (SECE) in piezoelectric energy harvesting by using impedance modeling *Smart Mater. Struct.* **28** 105053

[139] Li J, Zhou X, Huang G and Hu G 2016 Acoustic metamaterials capable of both sound insulation and energy harvesting *Smart Mater. Struct.* **25** 045013

[140] Yuan M, Cao Z, Luo J and Ohayon R 2018 Acoustic metastructure for effective low-frequency acoustic energy harvesting *J. Low Frequency Noise, Vib. Active Control* **37** 1015–29

[141] Chen Y, Huang G, Zhou X, Hu G and Sun C-T 2014 Analytical coupled vibroacoustic modeling of membrane-type acoustic metamaterials: membrane model *J. Acoust. Soc. Am.* **136** 969–79

[142] Chen Y, Huang G, Zhou X, Hu G and Sun C-T 2014 Analytical coupled vibroacoustic modeling of membrane-type acoustic metamaterials: plate model *J. Acoust. Soc. Am.* **136** 2926–34

[143] Hu G, Tang L and Cui X 2019 On the modelling of membrane-coupled Helmholtz resonator and its application in acoustic metamaterial system *Mech. Syst. Signal Process.* **132** 595–608

[144] Wang X, Xu J, Ding J, Zhao C and Huang Z 2019 A compact and low-frequency acoustic energy harvester using layered acoustic metamaterials *Smart Mater. Struct.* **28** 025035

[145] Zhang X, Zhang H, Chen Z and Wang G 2018 Simultaneous realization of large sound insulation and efficient energy harvesting with acoustic metamaterial *Smart Mater. Struct.* **27** 105018

[146] Guo H, Wang Y, Wang X and Xu C 2018 Investigation on acoustic energy harvesting based on quarter-wavelength resonator phononic crystals *Adv. Mech. Eng.* **10** 1687814017748077

[147] Jin M, Liang B, Yang J, Yang J and Cheng J-C 2019 Ultrathin planar metasurface-based acoustic energy harvester with deep subwavelength thickness and mechanical rigidity *Sci. Rep.* **9** 1–7

[148] Sun K H, Kim J E, Kim J and Song K 2017 Sound energy harvesting using a doubly coiled-up acoustic metamaterial cavity *Smart Mater. Struct.* **26** 075011
[149] Fan L, He Y, Chen X-A and Zhao X 2019 Acoustic energy harvesting based on the topological interface mode of 1D phononic crystal tube *Appl. Phys. Express* **13** 017004
[150] Pal R K and Ruzzene M 2017 Edge waves in plates with resonators: an elastic analogue of the quantum valley hall effect *New J. Phys.* **19** 025001
[151] Apigo D J, Cheng W, Dobiszewski K F, Prodan E and Prodan C 2019 Observation of topological edge modes in a quasiperiodic acoustic waveguide *Phys. Rev. Lett.* **122** 095501
[152] Yuan M, Cao Z, Luo J and Chou X 2019 Recent developments of acoustic energy harvesting: a review *Micromachines* **10** 48
[153] Hu G, Tang L, Das R and Marzocca P 2018 A two-degree-of-freedom piezoelectric energy harvester with stoppers for achieving enhanced performance *Int. J. Mech. Sci.* **149** 500–7
[154] Tang L, Yang Y and Soh C K 2010 Toward broadband vibration-based energy harvesting *J. Intell. Mater. Syst. Struct.* **21** 1867
[155] Wu Y, Ji H, Qiu J and Han L 2017 A 2-degree-of-freedom cubic nonlinear piezoelectric harvester intended for practical low-frequency vibration *Sens. Actuators* A **264** 1–10
[156] Liu W, Yuan Z, Zhang S and Zhu Q 2019 Enhanced broadband generator of dual buckled beams with simultaneous translational and torsional coupling *Appl. Energy* **251** 113412
[157] Fang X, Wen J, Yin J and Yu D 2016 Wave propagation in nonlinear metamaterial multi-atomic chains based on homotopy method *AIP Adv.* **6** 121706
[158] Fang X, Wen J, Bonello B, Yin J and Yu D 2017 Ultra-low and ultra-broad-band nonlinear acoustic metamaterials *Nat. Commun.* **8** 1288
[159] Khajehtourian R and Hussein M 2013 Nonlinear locally resonant metamaterials: modeling and dispersion characteristics *Proc. Phononics* 180–1
[160] Hwang M and Arrieta A F 2019 Energy harvesting characteristics in metamaterials based on bistable lattices *Active and Passive Smart Structures and Integrated Systems XIII* (International Society for Optics and Photonics)
[161] Wu F, Hou Z, Liu Z and Liu Y 2001 Point defect states in two-dimensional phononic crystals *Phys. Lett.* A **292** 198–202
[162] Wu L-Y, Chen L-W and Liu C-M 2009 Acoustic pressure in cavity of variously sized two-dimensional sonic crystals with various filling fractions *Phys. Lett.* A **373** 1189–95
[163] Wu L-Y, Chen L-W and Liu C-M 2009 Acoustic energy harvesting using resonant cavity of a sonic crystal *Appl. Phys. Lett.* **95** 013506
[164] Yang A, Li P, Wen Y, Lu C, Peng X, Zhang J and He W 2013 Enhanced acoustic energy harvesting using coupled resonance structure of sonic crystal and Helmholtz resonator *Appl. Phys. Express* **6** 127101
[165] Yang A, Li P, Wen Y, Lu C, Peng X, Zhang J and He W 2014 Enhanced acoustic wave localization effect using coupled sonic crystal resonators *Appl. Phys. Lett.* **104** 151904
[166] Yang A, Li P, Wen Y, Yang C, Wang D, Zhang F and Zhang J 2015 High-Q cross-plate phononic crystal resonator for enhanced acoustic wave localization and energy harvesting *Appl. Phys. Express* **8** 057101
[167] Carrara M, Cacan M, Toussaint J, Leamy M, Ruzzene M and Erturk A 2013 Metamaterial-inspired structures and concepts for elastoacoustic wave energy harvesting *Smart Mater. Struct.* **22** 065004
[168] Lv H, Tian X, Wang M Y and Li D 2013 Vibration energy harvesting using a phononic crystal with point defect states *Appl. Phys. Lett.* **102** 034103
[169] Carrara M, Cacan M, Toussaint J, Leamy M, Ruzzene M and Erturk A 2012 Metamaterial concepts for structure-borne wave energy harvesting: focusing, funneling, and localization *ASME 2012 Conf. on Smart Materials, Adaptive Structures and Intelligent Systems* (American Society of Mechanical Engineers Digital Collection)
[170] Aly A H, Nagaty A, Khalifa Z and Mehaney A 2018 The significance of temperature dependence on the piezoelectric energy harvesting by using a phononic crystal *J. Appl. Phys.* **123** 185102
[171] Aly A H, Nagaty A and Mehaney A 2018 Thermal properties of one-dimensional piezoelectric phononic crystal *Eur. Phys. J.* B **91** 251
[172] Jo S-H, Yoon H, Shin Y C, Kim M and Youn B D 2020 Elastic wave localization and harvesting using double defect modes of a phononic crystal *J. Appl. Phys.* **127** 164901
[173] Oudich M and Li Y 2017 Tunable sub-wavelength acoustic energy harvesting with a metamaterial plate *J. Phys. D: Appl. Phys.* **50** 315104
[174] Jo S-H, Yoon H, Shin Y C, Choi W, Park C-S, Kim M and Youn B D 2020 Designing a phononic crystal with a defect for energy localization and harvesting: supercell size and defect location *Int. J. Mech. Sci.* **179** 105670
[175] Park C-S, Shin Y C, Jo S-H, Yoon H, Choi W, Youn B D and Kim M 2019 Two-dimensional octagonal phononic crystals for highly dense piezoelectric energy harvesting *Nano Energy* **57** 327–37
[176] Hu H, Zheng Y, Jiang S, Hu Y and Laude V 2017 Piezoelectric harvester scavenges energy from cavity of phononic crystal *2017 Joint Conf. European Frequency and Time Forum and IEEE Int. Frequency Control Symp. (EFTF/IFCS)* (IEEE)
[177] Geng Q, Cai T and Li Y 2019 Flexural wave manipulation and energy harvesting characteristics of a defect phononic crystal beam with thermal effects *J. Appl. Phys.* **125** 035103
[178] Deng T, Zhang S and Gao Y 2019 Tunability of band gaps and energy harvesting based on the point defect in a magneto-elastic acoustic metamaterial plate *Appl. Phys. Express* **13** 015503
[179] Deng T, Zhang S and Gao Y 2019 A magnetic-dependent vibration energy harvester based on the tunable point defect in 2D magneto-elastic phononic crystals *Crystals* **9** 261
[180] Carrara M, Cacan M, Leamy M, Ruzzene M and Erturk A 2012 Dramatic enhancement of structure-borne wave energy harvesting using an elliptical acoustic mirror *Appl. Phys. Lett.* **100** 204105
[181] Tol S, Vora F T, Degertekin F L and Erturk A 2016 Embedded elastic wave mirrors for enhanced energy harvesting *2016 IEEE SENSORS* (IEEE)
[182] Tol S, Degertekin F and Erturk A 2017 Structurally embedded reflectors and mirrors for elastic wave focusing and energy harvesting *J. Appl. Phys.* **122** 164503
[183] Darabi A and Leamy M J 2017 Analysis and experimental verification of multiple scattering of acoustoelastic waves in thin plates for enhanced energy harvesting *Smart Mater. Struct.* **26** 085015
[184] Qi S and Assouar B 2017 Acoustic energy harvesting based on multilateral metasurfaces *Appl. Phys. Lett.* **111** 243506
[185] Tol S, Degertekin F L and Erturk A 2016 Gradient-index phononic crystal lens-based enhancement of elastic wave energy harvesting *Appl. Phys. Lett.* **109** 063902
[186] Tol S, Degertekin F L and Erturk A 2016 Dramatic enhancement of elastic wave energy harvesting using a

gradient-index phononic crystal lens *ASME 2016 Conf. on Smart Materials, Adaptive Structures and Intelligent Systems* (American Society of Mechanical Engineers Digital Collection)

[187] Tol S, Degertekin F and Erturk A 2017 3D-printed lens for structure-borne wave focusing and energy harvesting *Active and Passive Smart Structures and Integrated Systems 2017* (International Society for Optics and Photonics)

[188] Tol S, Degertekin F and Erturk A 2019 3D-printed phononic crystal lens for elastic wave focusing and energy harvesting *Addit. Manuf.* **29** 100780

[189] Hyun J, Choi W and Kim M 2019 Gradient-index phononic crystals for highly dense flexural energy harvesting *Appl. Phys. Lett.* **115** 173901

[190] Zareei A, Darabi A, Leamy M J and Alam M-R 2018 Continuous profile flexural GRIN lens: focusing and harvesting flexural waves *Appl. Phys. Lett.* **112** 023901

[191] Yi K, Monteil M, Collet M and Chesné S 2017 Smart metacomposite-based systems for transient elastic wave energy harvesting *Smart Mater. Struct.* **26** 035040

[192] Tol S, Degertekin F L and Erturk A 2017 Low-frequency elastic wave focusing and harvesting via locally resonant metamaterials *ASME 2017 Conf. on Smart Materials, Adaptive Structures and Intelligent Systems* (American Society of Mechanical Engineers Digital Collection)

[193] Tol S, Degertekin F and Erturk A 2017 Phononic crystal Luneburg lens for omnidirectional elastic wave focusing and energy harvesting *Appl. Phys. Lett.* **111** 013503

[194] Li K and Rizzo P 2015 Energy harvesting using arrays of granular chains and solid rods *J. Appl. Phys.* **117** 215101

[195] Kim E and Yang J 2019 Review: wave propagation in granular metamaterials *Funct. Compos. Struct.* **1** 012002

[196] Li K, Rizzo P and Bagheri A 2015 A parametric study on the optimization of a metamaterial-based energy harvester *Smart Mater. Struct.* **24** 115019

[197] Rizzo P and Li K 2017 Analysis of the geometric parameters of a solitary waves-based harvester to enhance its power output *Smart Mater. Struct.* **26** 075004

[198] Li K and Rizzo P 2017 Experimental parametric analysis of an energy harvester based on highly nonlinear solitary waves *J. Intell. Mater. Syst. Struct.* **28** 772–81

[199] Lumentut M and Shu Y 2018 A unified electromechanical finite element dynamic analysis of multiple segmented smart plate energy harvesters: circuit connection patterns *Acta Mech.* **229** 4575–604

[200] Lien I and Shu Y 2012 Multiple piezoelectric energy harvesters connected to different interface circuits *Active and Passive Smart Structures and Integrated Systems 2012* (International Society for Optics and Photonics)

[201] Hu G, Tang L, Das R and Aw K 2018 Internally coupled piezoelectric metamaterial beam with multi-functionalities *Active and Passive Smart Structures and Integrated Systems XII* (International Society for Optics and Photonics)

[202] Mir F, Saadatzi M, Ahmed R U and Banerjee S 2018 Acoustoelastic MetaWall noise barriers for industrial application with simultaneous energy harvesting capability *Appl. Acoust.* **139** 282–92

[203] Assouar B, Oudich M and Zhou X 2015 Sound insulation and energy harvesting based on acoustic metamaterial plate *Health Monitoring of Structural and Biological Systems 2015* (International Society for Optics and Photonics)

[204] Ma K-J, Tan T, Liu F-R, Zhao L-C, Liao W-H and Zhang W-M 2020 Acoustic energy harvesting enhanced by locally resonant metamaterials *Smart Mater. Struct.* **29** 075025